Object Tracking Using Color Multiset Coding (CMC)

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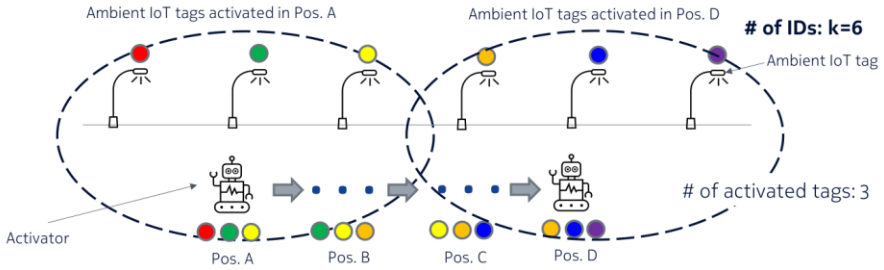
Background

Ambient IoT



5G/6G wireless IoT

- 3GPP NB-IoT/eMTC and NR RedCap
- Low cost and low power devices for wide area IoT
- Many applications require battery-less devices

Ambient tag

- Device that modulates an incoming signal with some information (e.g., ID of the tag) which is conveyed to a receiving node via backscatter reflection
- Use scenarios (conventional cellular IoT, RFID, 6G backscatter IoT)
- Application examples: utility metering, environmental sensing, machine-specific monitoring, inventory, warehouse

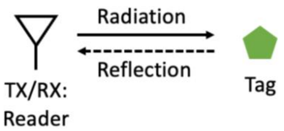
A. Conventional cellular IoT: Active radio

B. Conventional RFID: Backscatter radio, monostatic architecture

C. 6G B-IoT proposal: Backscatter radio, multi-static architecture

Robot positioning and object tracking

Use IDs composed of fewer bits is preferred



A positioning system where a robot determines its position from <u>local</u> observations

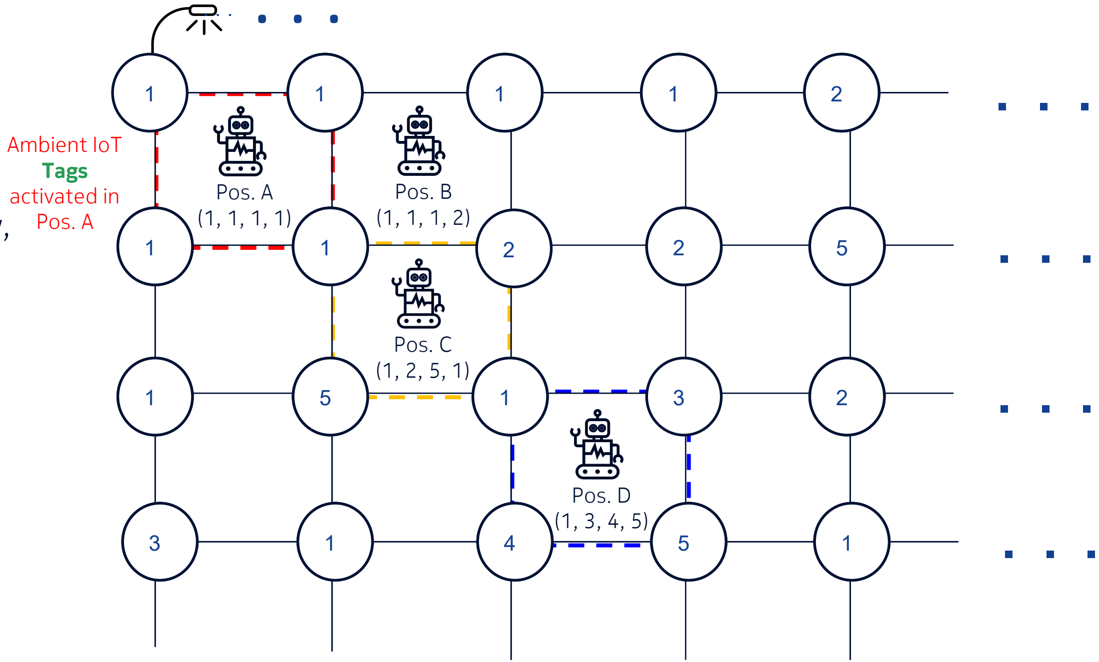
- Practical relevance
- Mathematical interest

<u>Limited</u> observational capabilities

- The robot (or system) does not have access to the full pattern of letters/IDs in the window, but only to the no. of occurrences of each (their cardinality or intensity)

Tag letters: (IDs)

1	1	1	1	2	2	2
1	1	2	2	5	2	2
1	5	1	3	2	4	2
3	1	4	5	1	4	2
3	1	3	2	4	5	4
3	3	5	3	3	4	4
3	3	3	4	4	4	4



Multiset vs. ordered pattern

Tag letters:
(IDs)
k = 5

1	1	1	1	2	2	2
1	1	2	2	5	2	2
1	5	1	3	2	4	2
3	1	4	5	1	4	2
3	1	3	2	4	5	4
3	3	5	3	3	4	4
3	3	3	4	4	4	4

Measuring an ordered pattern requires sophisticated hardware

- Directional antenna (radio signal)
- Camera (instead of simple photodiode in light system)

Requirement: no two windows contain the same multiset of letters

- {1, 1, 2, 5} would be considered as **equiv. to** {1, 1, 5, 2} as they are the same multiset, since they have the same no. of 1's and the same number of 2's (they have the same cardinality vector [2 1 0 0 1] for k = 5)

Measuring a multiset has lower implementation cost

- RFID tag, used in retail for inventory management, self checkout, object tracking
- Cost increases with the number of tag identifiers and intensity levels or colors

Basic solution

Tag ID is denoted/represented by a letter (or color)

The activator (object)

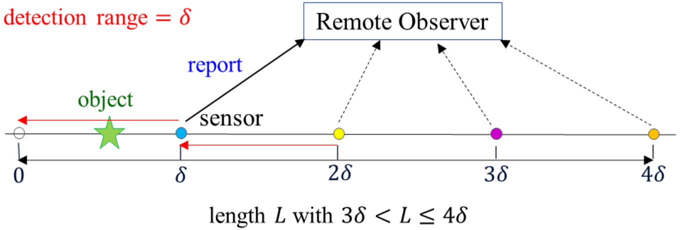
- Can randomly appear on a line (or a grid in 2D)
- We want to determine the position of the <u>object</u> and rely on sensors that can detect objects with a predefined range and report the observations to a remote observer

Each tag (sensor) has its ID

- Sensors are equipped with a transmitter that can transmit at a <u>limited</u> data rate R
- Once triggered, it transmits its own ID to inform
- The no. of tags in a factory can be <u>large</u> (> 10^4) such that the no. of bits used to represent each ID would be large

Straightforward protocol

- Label each sensor with different ID, however this would require a large no. of unique IDs and many bits to represent or larger R



Multiset Combinatorial Gray Code (MCGC) based

Color Coding Protocol

Multiset color coding in reusing a much smaller number of IDs with chosen system parameter $m\delta$ (m = 1, 2, 3, ...)

- To require the minimal no. of bits for labeling each sensor
- To determine the factor of reduction on the no. of IDs (resource utilization efficiency)
- Optimal constructions for the required sequence or code and the generalization for systems of higher dimension
- To derive the upper and lower bounds for the maximum system size

Basic solution

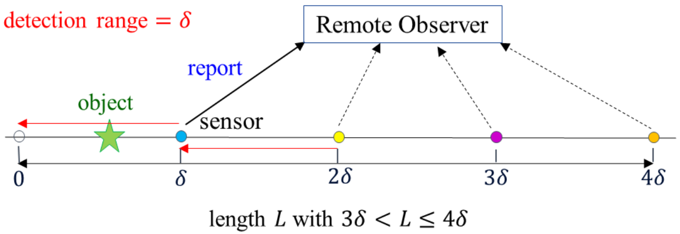


Fig. 1. $N = 4$ sensors are sufficient for the case that $3\delta < L \leq 4\delta$. Sensor at $(j+1)\delta$ detects whether the object is located within $[j\delta, (j+1)\delta)$, where $j = 0, 1, 2, 3$, and transmit its preassigned unique ID to signal (report) to a remote observer.

vs.

New scheme

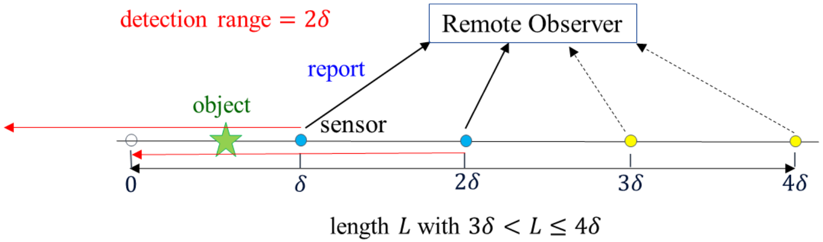


Fig. 2. We use $N = 4$ sensors in this example. As $m = 2$, if the object is located within $[j\delta, (j+1)\delta)$, the sensors at $(j+1)\delta$ and $(j+2)\delta$ are triggered. Both of them report the detection of object to the remote observer. As shown above, only two IDs (indicated as two colors) are required for the four sensors (instead of four IDs spent in the case of Fig. 1) since the remote observer can distinguish where the object is located by each set of m consecutive sensors.

Related work

Literature

Universal cycles/torii, Eulerian circuits [Chung-Graham-Diaconis 1992]

- Cyclic sequence of symbols or letters whose consecutive substrings (fixed length) to represent each object
- Each window (length m) outputs a function of its **content**
- Each possible output appears exactly once (no output repetition in maximizing the network or system size L)

Universal cycles for multisets

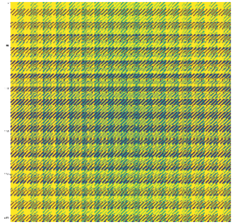
- Open problem [Knuth 2011]
- Some interesting progress [Hurlbert-Johnson-Zahl 2009, Blanca-Godbole 2011]

Universal cycles for set

- Conjecture in [Chung-Diaconis-Graham 1992]
- Some result and discussion by [Glock-Joos-Kuhn 2020]

de Bruijn torii and M-sequences

- Can trace back to [MacWilliams-Sloane '76, Kumar-Wei '92, Hurlbert-Isaak '93,'95, Hurlbert-Mitchell-Paterson'96]



2D example: L=nxn=256, window size = mxm=8, #color(s) k=5

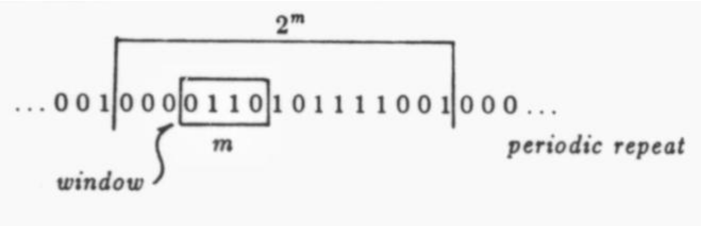
[C. S. Chen, P. Keevash, W. S. Kennedy, E. de Panafieu, and A. Vetta, "Robot Positioning Using Torus Packing for Multisets," 51st EATCS International Colloquium on Automata, Languages and Programming (ICALP), Tallinn, 2024]

Example (1/2)

Some literature

de Bruijn sequence

- See every possible m-tuple exactly once



A de Bruijn Sequence for $m = 4$.

Each possible output appears exactly once (no output repetition in <u>maximizing</u> the network or system size L), k = 2:
{0,0,0,0}, {0,0,0,1}, {0,0,1,1}, {0,1,1,0}, {1,0,1,0}, {0,1,0,1}, {1,0,1,1}, {0,1,1,1}, {1,1,1,1}, …, {1,0,0,1}

Multiset
[4,0], [3,1], [2,2], [2,2], [2,2], [2,2], …

2-dimensional array

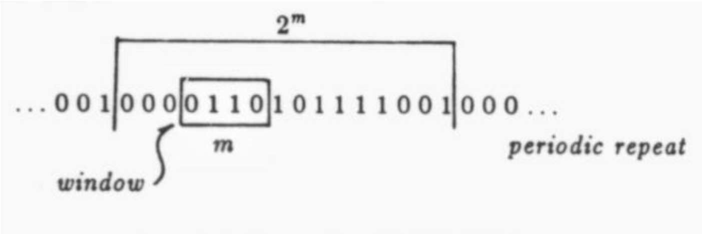
A Pseudo-Random Array for $M = 9$, $N = 7$, $m = 2$, $n = 3$.

2D example: L=9x7=63, window size = mxn=2x3=6, #color(s) k=2

Example (2/2)

Some literature

de Bruijn sequence



A de Bruijn Sequence for $m = 4$.

Each possible output appears exactly once (no output repetition in maximizing the network or system size L), k = 2:
{0,0,0,0}, {0,0,0,1}, {0,0,1,1}, {0,1,1,0}, {1,0,1,0}, {0,1,0,1}, {1,0,1,1}, {0,1,1,1}, {1,1,1,1}, ..., {1,0,0,1}

Multiset
[4,0], [3,1], [2,2], [2,2], [2,2], [2,2], ...

MCGC: No two windows contain the same multiset of letters (the order of the elements does not matter)

1D: L = 22, m = 3, we can use k = 4 letters to complete the grid

2 2 1 1 1 3 3 3 4 4 4 2 2 2 3 4 1 4 1 2 3 3

2D: L = MxN = 8x8 2, k = 5

1	1	1	1	3	4	2	2
1	1	2	5	1	4	2	2
2	2	2	2	5	4	4	2
4	3	5	5	5	4	5	2
3	5	1	5	5	5	2	1
3	1	1	5	3	3	3	3
3	3	1	4	5	3	4	4
3	3	1	2	4	4	4	4

Problem Statement

A **grid** of size N x M (e.g., 8 x 8 if it is a square)

We use a set of colors or **letters** {`1', `2', `3', ... } to mark each dot or **pixel**

We will check each **block** (e.g., size 2 x 2) that how many 1's, 2's, 3's, ... there are in each block. We can represent these numbers by a **vector** (denotes the cardinality of each color that we use in a block)

We wish each block would have **distinct** cardinality vector

The color vector of a block is denoted by $(x_1, x_2, \ldots, x_k)$

We can prove that using 4 colors won't work.

Proof : There are in total 49 blocks (of size 2 x 2) that wish to have **distinct** cardinality vector. By counting (permutation with replacement, a.k.a. ***combination with repetition***) on grid 8 x 8, the no. of combinations when using only 4 colors = (4+4-1)! / (4! *(4-1)!) = 35, which is less than 49.

block (or window) size n = 2x2 = 4

1	1	1	1	3	4	2	2
1	1	2	5	1	4	2	2
2	2	2	2	5	4	4	2
4	3	5	5	5	4	5	2
3	5	1	5	5	5	2	1
3	1	1	5	3	3	3	3
3	3	1	4	5	3	4	4
3	3	1	2	4	4	4	4

grid

A coding of the grid is proper if the color vectors of all blocks are **distinct**, where $x_1 + x_2 + \ldots + x_k = n$

$$C(k, n) = \binom{n + k - 1}{n}$$

Reference: R. A. Brualdi, Introductory Combinatorics. Pearson Prentice Hall, 2010.

How would you fill each pixel?

Best effort (greedy algorithm)

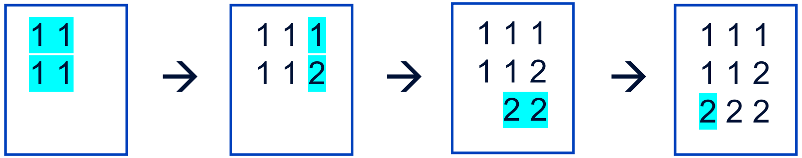
- May lead to very sub-optimal solution (using too many colors or inefficient resource utilization)

Constraint satisfying problem (CSP) standard solver

- Very high computational time complexity (empirically unable to solve problem of large size)

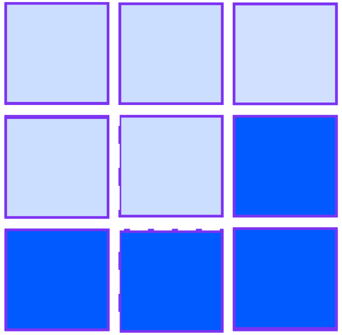
Example

- Problem of size 3x3 (the grid), which contains four blocks of each of size 2x2
- Easy: two colors {‘1’, ‘2’} are sufficient, see below



Grid: 3 x 3
Each block: 2 x 2

Done, but what if grid of 10^3x10^3?



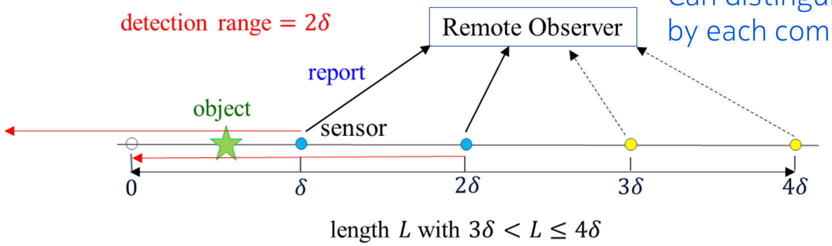
[N. Creignou, S. Khanna, and M. Sudan, *Complexity Classifications of Boolean Constraint Satisfaction Problems*, Society for Industrial & Applied Math., 2001]

Multiset combinatorial Gray coding

A generalization of binary reflected Gray code

To use a much smaller no. of IDs with a **system configuration parameter** $m\delta$ (m = 1, 2, 3, ...)

- Once L, δ and m are given, it suffices to find k such that $N \leq N_m(k)$, where N_m is the no. of combinations of the IDs of m consecutive sensors
- One can reduce the no. of required bits for each ID by a **factor of 1/m** by taking the advantage of their possible combinations, good (necessary) for supporting large system
- Determine the upper and lower **bounds** for the maximum system size
- Thanks to **coding**, we can use a minimal k and require the minimal no. of bits for labeling each sensor



Can distinguish where it is located by each combination: {BB}, {BY}, {YY}

For **accuracy** δ, set a sensor at x to detect object in the range $[x - m\delta, x)$, where m is positive integer, while sensors are located at δ, 2δ, ..., $N\delta$, where $N = \lceil L/\delta \rceil$

Construction and bounds

Some technical results

Upper bound

For given positive integers m (= 1, 2, 3, …) and k, one has:

$$\boxed{N_m(k) \leq \binom{k+m-1}{m} + m - 1}$$

Proof. Let $S = s_0 s_1 \cdots s_{N-1}$ is a longest m-distinguishable sequence. By definition, the $N - m + 1$ multisets $S_t(m)$, $0 \leq t \leq N - m$, are all distinct. By representing each of these multiset as $\{1^{e_1}, \ldots, k^{e_k}\}$, where e_s indicates the multiplicity of the element s, those non-negative multiplicities must satisfy $e_1 + e_2 + \cdots + e_k = m$. Therefore, the number of all possible $S_t(m)$ is $\binom{k+m-1}{m}$, the number of non-negative integral solutions of the equation $e_1 + e_2 + \cdots + e_k = m$. It follows that $N - m + 1 \leq \binom{k+m-1}{m}$. $\square$

(Example: 111222333 is a cyclic 3-distinguishable sequence)

Lower bound

Corollary 4. *Let $m \in \mathbb{N}$ and k be a sufficiently large integer. Then,*

$$\boxed{N_m(k) \geq \binom{k}{m} + m - 1}$$

if m divides $\binom{k-1}{m-1}$; and, there exists a constant c such that

$$N_m(k) \geq \binom{k}{m} - ck + m - 1$$

otherwise.

[C. S. Chen, Y.-H. Lo, W. S. Wong, and Y. Zhang, "Object Tracking Using Multiset Color Coding," *submitted to* Intl. Symp. on Information Theory and Its Applications, 2024]

Construction and bounds

A construction solution for any even k ≥ 2, m = 2, 3

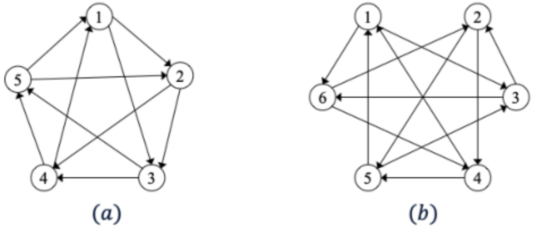
Let $\mathbf{G_k}$ denote the complete **graph** of k vertices labeled by elements in {1, 2, ..., k}. An **Eulerian circuit** of a graph is a circuit that contains all edges (e.g., Eulerian circuit of G_5 is a circuit with travel 1234513524)

Let **F** be a 1-factor in G_k, i.e., a collection of k/2 independent edges (e.g., {1,2}, {3,4}, {5,6} for G_6)

- Step (i) Find an Eulerian circuit in G_k − F (e.g., 132451625364 shown by Fig. (b))
- Step (ii) Repeat **the first occurrence of every element** and denote the obtained cyclic sequence by S
- Step (iii) Pick an arbitrary **t-cut** of S (e.g., choose t = 0)
  - To do: "cut" S at position t and repeat the consequent m−1 elements

    i.e., "113322445516625364" + "1"

$$132451625364 \xrightarrow{\text{(ii)}} 113322445516625364$$
$$\xrightarrow{\text{(iii)}} 1133224455166253641$$



(a) An Eulerian circuit of G_5. The vertices that it travels is 1234513524 in order. (b) An Eulerian circuit of $G_6 - F$, where F contains the three edges $\{1,2\}, \{3,4\}$ and $\{5,6\}$. The vertices that it travels is 132451625364 in order.

[C. S. Chen, Y.-H. Lo, W. S. Wong, and Y. Zhang, "Object Tracking Using Multiset Color Coding," *submitted to* Intl. Symp. on Information Theory and Its Applications, 2024]

Color coding gain

Factor of reduction

Corollary 4. *Let $m \in \mathbb{N}$ and k be a sufficiently large integer. Then,*

$$N_m(k) \geq \binom{k}{m} + m - 1$$

Let m be a fixed positive integer. For large enough k, by Corollary 4, $\boxed{N \approx \frac{k^m}{m!}}$

Then, we have

$$\begin{aligned} \log_2 N &\approx m \log_2 k - \log_2 m! \\ &= m \log_2 k - (m \log_2 m - m \log_2 e + O(\log_2 m)), \end{aligned} \tag{8}$$

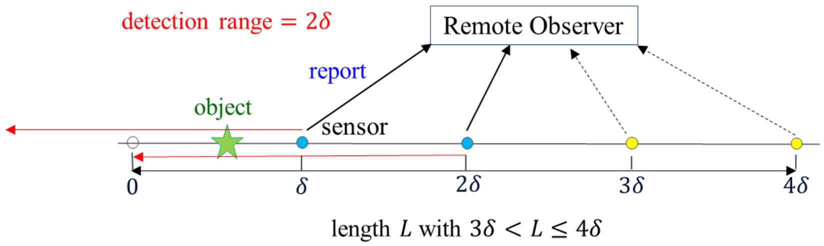
which implies that

$$\log_2 k \approx \frac{\log_2 N}{m} + \log_2 m - \log_2 e + \frac{c \log_2 m}{m}, \tag{9}$$

for some constant c. The identity in (8) is due to *Stirling's approximation formula* [18]. Therefore, by (9), the factor of reduction by the proposed color coding protocol is

$$\boxed{\frac{\log_2 k}{\log_2 N} = \frac{1}{m}.} \tag{10}$$

E.g., we can find k = 9 for N =164 (m = 3), the factor of reduction ≈ 0.43



Remark:
- For m > 1, one can consider as a sensing collaboration or cooperative localization, which employs a larger detection range but could offer the reduction factor to the no. of transmitted bits per node, as a kind of implementation tradeoff (a system parameter that one can choose)

[18] J. Dutka, "The early history of the factorial function," Archive for History of Exact Sciences, vol. 43, pp. 225–249, 1991.

Generalization to a 2D grid

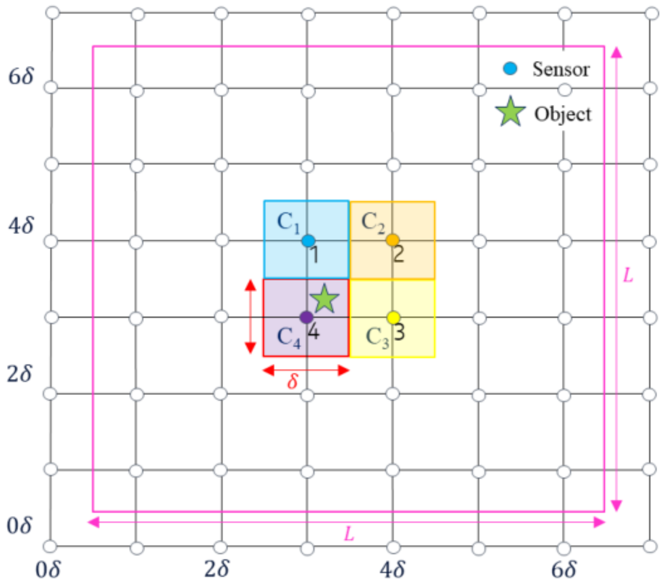


Fig. 1. $L = 6\delta$. Sensors are deployed in the grid topology to cover the monitored area $L \times L$, indicated by the magenta square. A sensor at $(i\delta, j\delta)$ would detect the presence of object in the vicinity $x \in ((i-0.5)\delta, (i+0.5)\delta]$ and $y \in ((j-0.5)\delta, (j+0.5)\delta]$, and then transmits the pre-assigned unique ID to signal. In the above example, sensor 4 is triggered and sends its ID, denoted by C_4, to inform a remote observer.

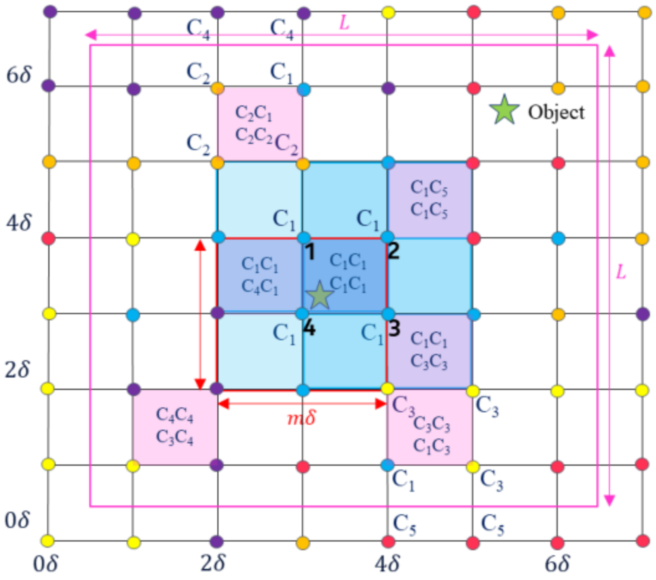


Fig. 2. $L = 6\delta$ and $m = 2$. Sensors are deployed for monitoring the same area with same size $L \times L$, indicated by the magenta square. A sensor at $(i\delta, j\delta)$ would detect the presence of object in the vicinity $x \in ((i-1)\delta, (i+1)\delta]$ and $y \in ((j-1)\delta, (j+1)\delta]$, and then transmits the pre-assigned ID to signal. Here, sensors 1, 2, 3 and 4 are triggered as $m = 2$. In contrary to the setup in Fig. 1, we do not have to apply distinct ID for each sensor. As shown above, only 5 distinct IDs (indicated as 5 colors) are required for the sensors since the remote observer can distinguish where the object is located by each set of m^2-neighboring sensor IDs.

  Public

Summary and future work

A newly defined protocol using MCGC to minimize the no. of required distinct IDs

A natural and interesting problem of practical importance

- Tracking an object or determining the position of a robot from local observations with limited capability
- Simple and efficient solution at lower cost and system complexity
- Possible use cases: industrial IoT, ambient IoT, smart cities, modern warehouse, B5G

Random coloring is known inefficient (birthday paradox)

- Optimal constructions of MCGC and the generalization to higher dimension
- General result and upper/lower bounds
- Disk or other window shape instead of square or rectangle

Error correction

- Real-world factors or challenges such as interference and measurement error

Could neural networks or similar learn to build suitable torus packing or codeword for the MCGC?

- Machine learning
- Deep learning

NOKIA