The Doppler-Shannon Paradigm for Vehicular Networks

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Objectives

1. Doppler aware design and performance evaluation of vehicular/ non-terrestrial networks using stochastic geometry
    - Statistical characterization of the Doppler shift experienced at the UE (in 2D, 3D, spherical coordinates)
    - Understanding time-selectivity (coherence time) in wireless channels
    - Doppler aware UE association framework (comparison with classical shannon based association)
    - Performance evaluation

The Doppler Shift

- Static user equipment (UE) at the origin
- Mobile base station (BS), $\underline{v}$
- LoS between UE-BS, $\underline{r}$
- Angle made by LoS, θ
- Angle made by direction of motion (DoM), α
- The Doppler effect is caused by velocity component on the LoS: $\underline{v}_r$
- Doppler experienced at the UE

$$D = \frac{\underline{v}_r}{\lambda_w} = \frac{v\cos\gamma}{\lambda_w}. \qquad (1)$$

- $\lambda_w = \frac{c}{f_c}$, $\gamma =$ angle of interest $= (\theta - \alpha)$, c: speed of light, f_c: carrier frequency

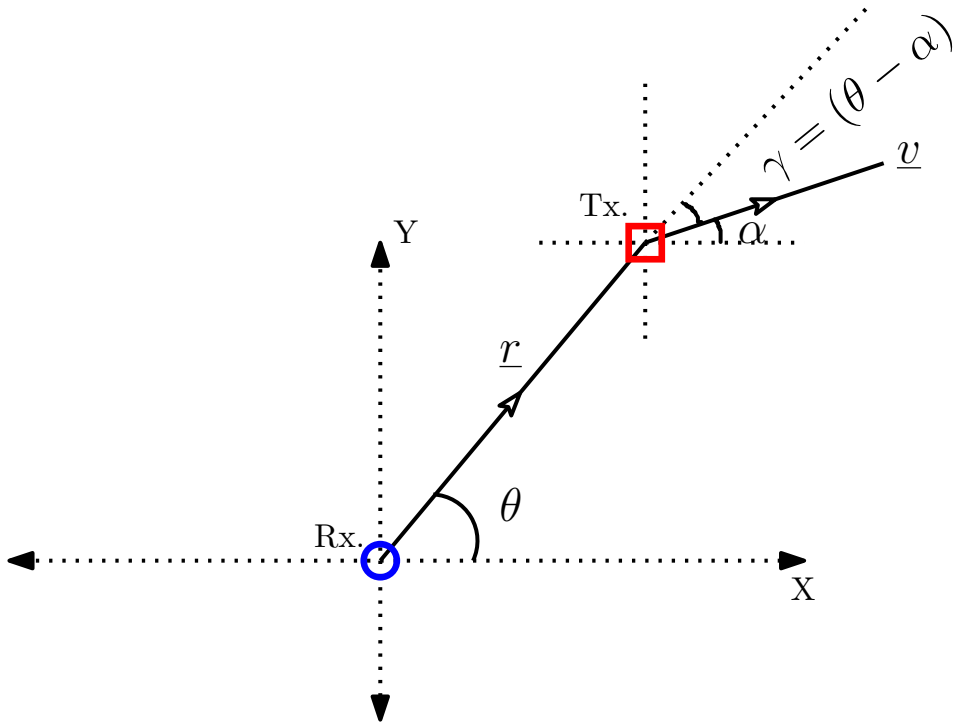


Figure 1: Considered system model.

Doppler Statistics

- The angle of interest γ, between the LoS $\underline{r}$ and the DoM $\underline{v}$ vectors is computed as

$$\begin{aligned}\gamma &= \cos^{-1}\left(\frac{\underline{r}\cdot\underline{v}}{rv}\right)\\ &= \cos^{-1}\left(\frac{rv\cos\theta\cos\alpha + rv\sin\theta\sin\alpha}{rv}\right)\\ &= \cos^{-1}(\cos(\theta-\alpha))\\ &= \pm(\theta-\alpha) = |(\theta-\alpha) \mod 2\pi|.\end{aligned} \tag{2}$$

- the Doppler shift experienced at the receiver is,

$$D = \frac{v}{\lambda_w}\cos(|\theta-\alpha|). \tag{3}$$

- For $\theta, \alpha \in U[0, 2\pi)$: $(\theta-\alpha) \sim \theta$ and statistically $\cos(\theta-\alpha) \sim \cos\theta$
- Hence, the Doppler is statistically independent of the DoM of the BS, α

- minimum Doppler: $\theta \to \pi/2 (\cos\theta \to 0)$; maximum Doppler: $\theta \to 0 (\cos\theta \to 1)$

Theorem

The distribution of the Doppler shift, in a 2D scenario, corresponds to a beta distribution given as,

$$f_D(\delta) = \frac{1}{\pi} \left(\frac{1}{\sqrt{\left(\frac{v}{\lambda_w}\right)^2 - \delta^2}} \right), \ \delta \in \left[\frac{-v}{\lambda_w}, \frac{v}{\lambda_w} \right]. \tag{4}$$

- $\frac{v}{\lambda_w}$: maximum Doppler (U shaped curve)
- The CDF can be derived as follows.

$$F_D(\delta) = \int_{-v/\lambda_w}^{\delta} \frac{1}{\pi} \frac{1}{\sqrt{\left(\frac{v}{\lambda_w}\right)^2 - x^2}} dx \tag{5}$$

$$= \frac{1}{\pi} \left(\sin^{-1} \left(\frac{x}{v/\lambda_w} \right) \right) \Bigg|_{x=-v/\lambda_w}^{\delta} \tag{6}$$

$$= \frac{1}{\pi} \sin^{-1} \frac{\delta \lambda_w}{v} + \frac{1}{2} \tag{7}$$

Coherence time (Time selectivity)

- Time duration for which the wireless channel remains "flat", i.e., statistically i.i.d.. In this duration, the wireless link is "stable" (no ISI).
- Coherence time is a function of the Doppler and is defined as, $T_C = \frac{k_1}{|D|}$, k_1 is an engineering parameter (constant)
- If the UE experiences high Doppler: coherence time is very small
- If UE experiences low Doppler: coherence time is large
- In vehicular networks: the coherence time can be no longer considered constant, it is a random variable
- Hence, an adaptive coherence time framework, capturing the Doppler effect, is needed to be studied

Engineering insights

$$
\begin{aligned}
T_C &= \frac{k_1}{|D|} \\
&= \frac{k_1 \lambda_w}{v|cos\theta|} \\
&= \frac{k_1 c}{v|\cos theta| f_c}
\end{aligned}
\tag{8}
$$

- Lower coherence time affects system performance
- Higher f_C: lower coherence time (high frequency bands cannot support high mobility)
- Higher v: lower coherence time

Coherence time statistics

CDF of magnitude of the Doppler, $|D|$:

$$F_{|D|}(w) = F_D(w) - F_D(-w) \tag{9}$$

$$= \frac{2}{\pi} \frac{1}{\sqrt{\left(\frac{v}{\lambda_w}\right)^2 - w^2}}, 0 \leq w < \frac{v}{\lambda_w} \tag{10}$$

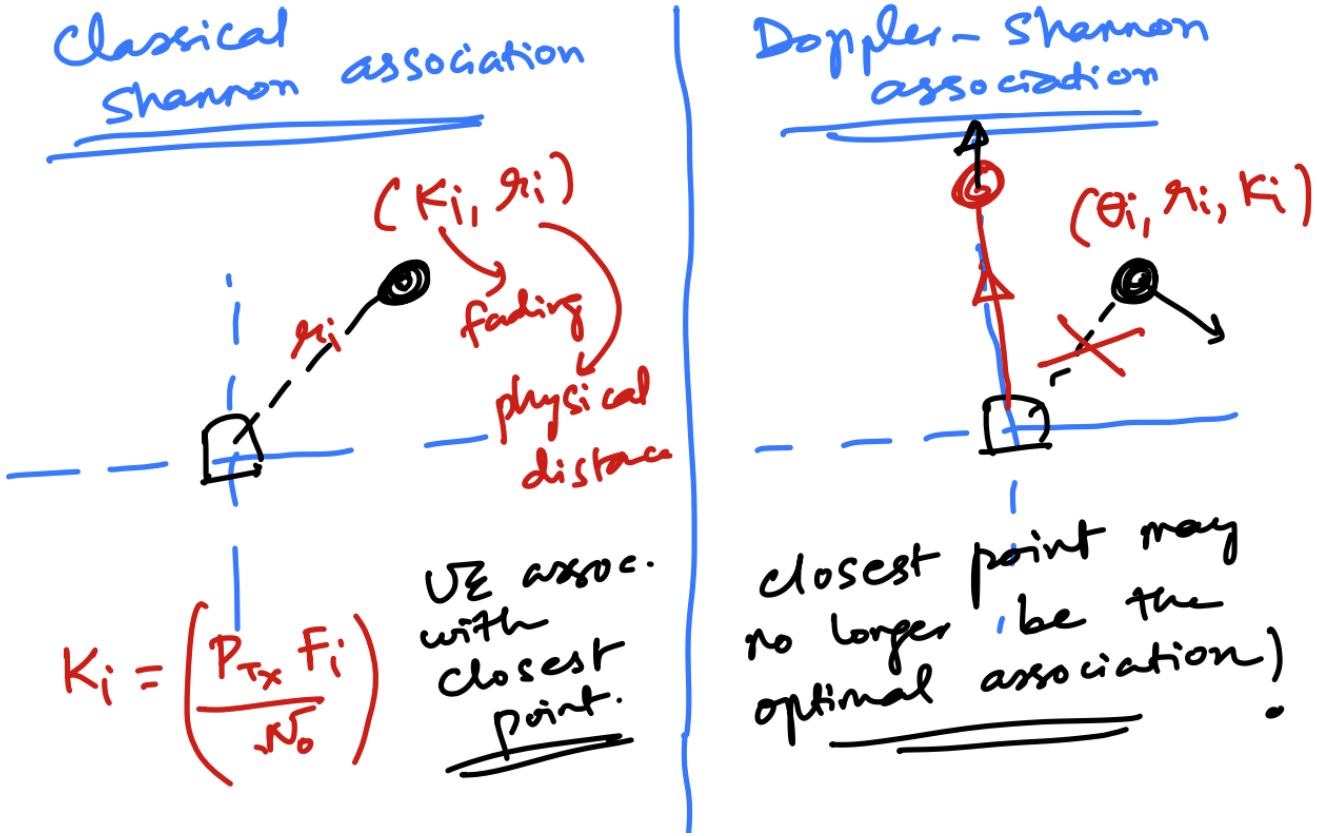
PDF of the coherence time:

$$f_{T_C}(y) = f_{|D|}(k_1/y) \left| \frac{\mathrm{d}|D|}{dy} \right| \tag{11}$$

$$= \frac{2}{\pi} \frac{1}{\sqrt{\left(\frac{v}{\lambda_w}\right)^2 - \left(\frac{k_1}{y}\right)^2}} \left(\frac{k_1}{y^2}\right), y \in [\frac{k_1 \lambda_w}{v}, \infty) \tag{12}$$

Doppler aware system design

- UE is static at the origin
- Poisson distributed points (mobile BS nodes), X_i
- $X_i = \{r_i, \theta_i\}$
- r_i: euclidean distance from the UE to the point
- θ_i: angle made from the LoS to the point
- K_i: fading channel coefficient at point X_i
- Classical Shannon association: (r_i, K_i)
- Doppler-Shannon association: (r_i, θ_i, K_i)



Doppler aware utilities - 1

$$U_i = Tc_i B log_2(1 + K_i r_i^{-\alpha}) \text{ [bits]} \tag{13}$$

Doppler aware UE association:

$$x^* = \arg\max_i U_i \tag{14}$$

[Failure metric]

$$\mathbb{P}(\max_i U_i(\theta, K, r) < \tau) = \exp\left(-\frac{\lambda}{2}\mathbb{E}[K^{2/\alpha}] \int_\theta \left(2^{\tau/(Tc(\theta)\times B)} - 1\right)^{-2/\alpha} \mathrm{d}\theta\right) \tag{15}$$

Success metric:

$$\int_{\tau=0}^{\infty} \left(1 - \exp\left(-\frac{\lambda}{2}\mathbb{E}[K^{2/\alpha}] \int_\theta \left(2^{\tau/(Tc(\theta)\times B)} - 1\right)^{-2/\alpha} \mathrm{d}\theta\right)\right) \mathrm{d}\tau \tag{16}$$

To add stability in the system design, as statistically, $Tc_i \to \infty$

$$U_i = \min\{Tc_i, \beta\} B log_2(1 + K_i r_i^{-\alpha}) \text{ [bits]} \tag{17}$$

where, $\beta = 10^4$ (big value), this parameter is to avoid $\theta = \pi/2$ and hence now $\theta \to \pi/2$

$$\mathbb{P}(\max_i U_i(\theta, K, r) < \tau) = \exp\left(-\frac{\lambda}{2}\mathbb{E}[K^{2/\alpha}] \int_\theta \left(2^{\tau/(\min\{Tc(\theta),\beta\} \times B)} - 1\right)^{-2/\alpha} \mathrm{d}\theta\right) \tag{18}$$

Classical Shannon UE association:

$$\mathbb{P}(\max_i U_i(K, r) < \tau) = \exp\left(-\frac{\lambda}{2} 2\pi\ \mathbb{E}[K^{2/\alpha}] \left(2^{\tau/(Tc \times B)} - 1\right)^{-2/\alpha}\right) \tag{19}$$

By Jensen's inequality (failure metric: Shannon >= Doppler Shannon)

$$\exp\left(-\frac{\lambda}{2} 2\pi\ \mathbb{E}[K^{2/\alpha}] \left(2^{\tau/(Tc \times B)} - 1\right)^{-2/\alpha}\right) \geq \exp\left(-\frac{\lambda}{2}\mathbb{E}[K^{2/\alpha}] \int_\theta \left(2^{\tau/(Tc(\theta) \times B)} - 1\right)^{-2/\alpha} \mathrm{d}\theta\right) \tag{20}$$

or, Shannon <= Doppler-Shannon [success-metric]

Expectation of fading

For a Nakagami fading environment, with shape parameter m and scale parameter Θ,

$$f_K(k) = \frac{1}{\Gamma(m)\Theta^m} k^{m-1} e^{-k/\Theta}, k > 0. \tag{21}$$

$$\mathbb{E}\left(K^{2/\alpha}\right) = \frac{1}{\Gamma(m)\Theta^m} \int_0^\infty k^{2/\alpha} k^{m-1} e^{-k/\Theta} \mathrm{d}k \tag{22}$$

$$\text{Substitute } u = k/\Theta \tag{23}$$

$$= \frac{\Theta^{2/\alpha}}{\Gamma(m)} \Gamma\left(m + \frac{2}{\alpha}\right) \tag{24}$$

for Rayleigh fading environment ($m = 1, \Theta = 1$),

$$K \sim \exp(1), f_K(k) = e^{-k}, k \geq 0 \tag{25}$$

$$\mathbb{E}\left[K^{2/\alpha}\right] = \int_0^\infty k^{2/\alpha} e^{-k} \mathrm{d}k \tag{26}$$

$$= \Gamma\left(1 + \frac{2}{\alpha}\right) \tag{27}$$

UE association with utility 1

For two BSs i, j, a UE at k associates with the BS from which it receives a better utility as shown below,

$$T_{C_{ik}} B \log_2 \left(1 + K_{ik} r_{ik}^{-\alpha}\right) \lessgtr T_{C_{jk}} B \log_2 \left(1 + K_{jk} r_{jk}^{-\alpha}\right). \tag{28}$$

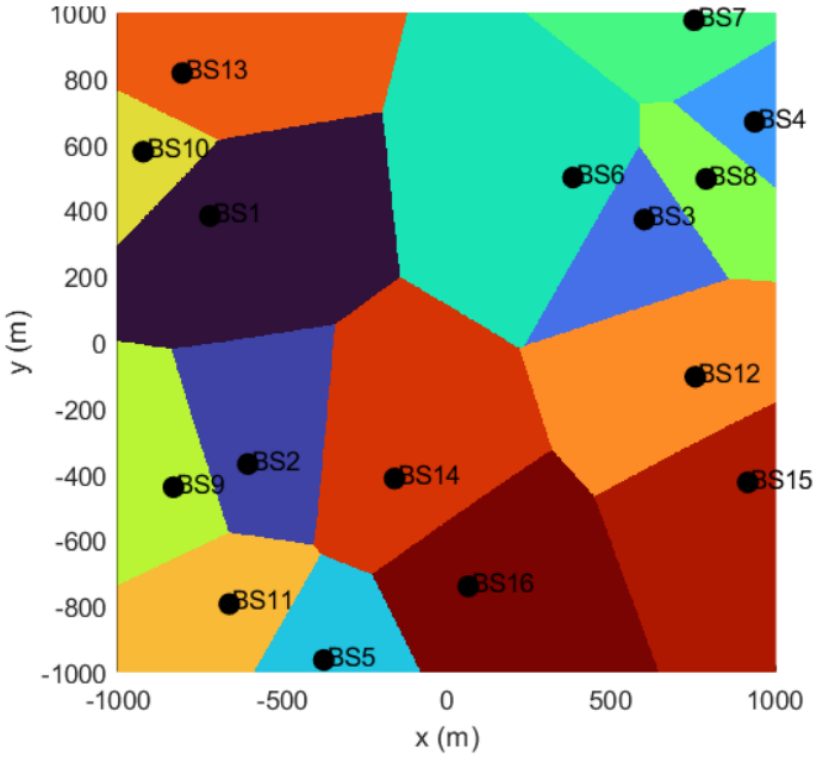


Figure 2: Classical shannon based association

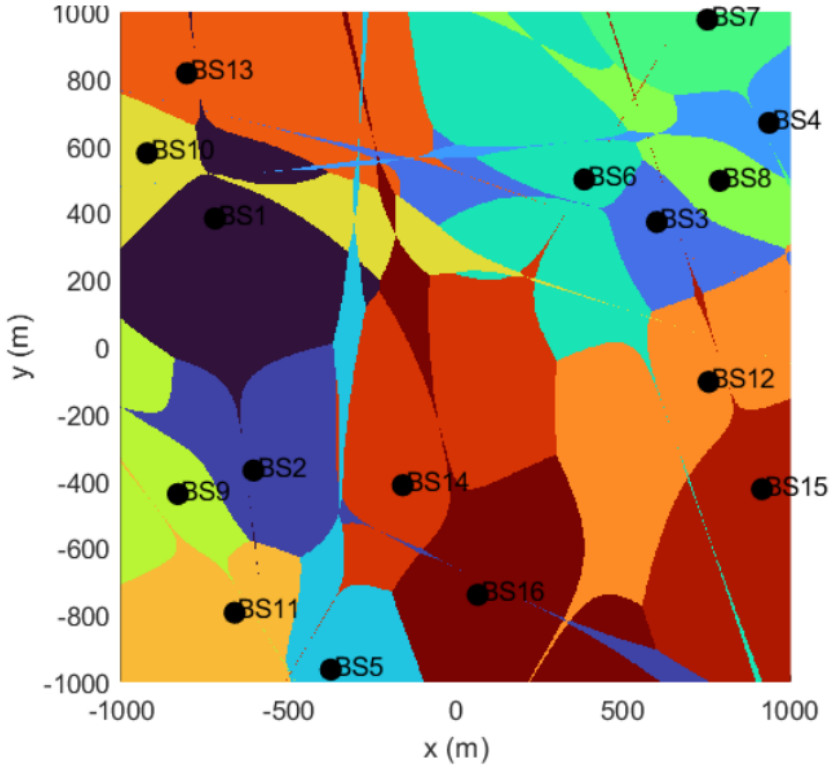
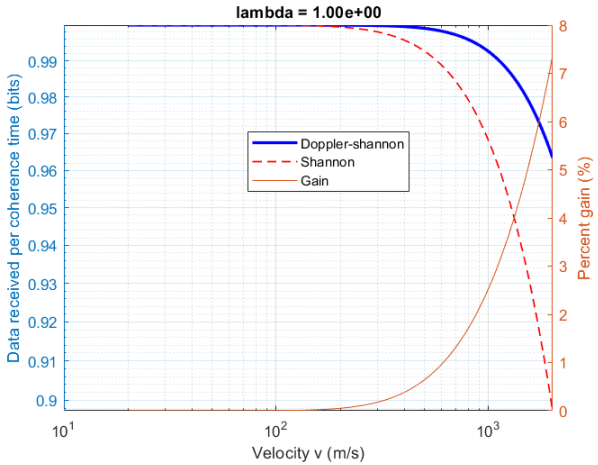
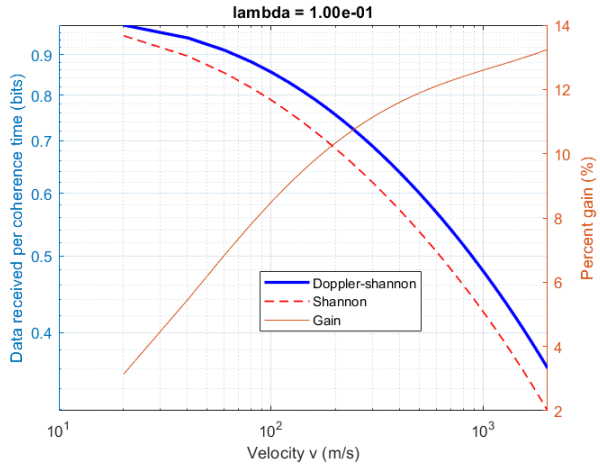


Figure 3: Doppler-Shannon association with utility 1

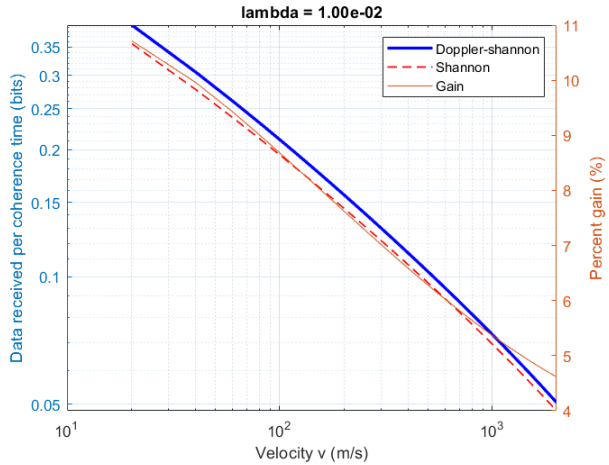
Performance evaluation of utility 1



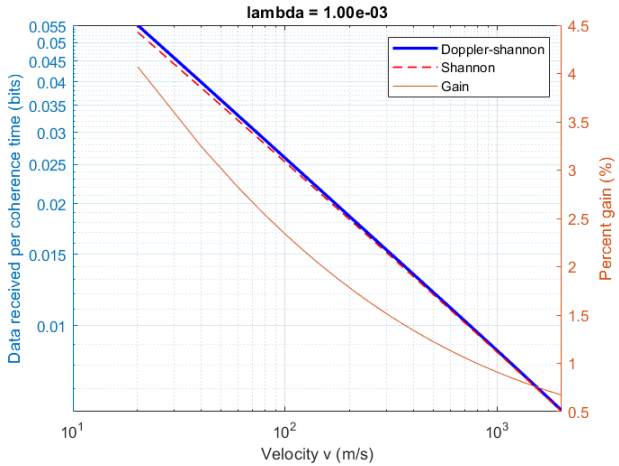
(a) $\lambda = 1$



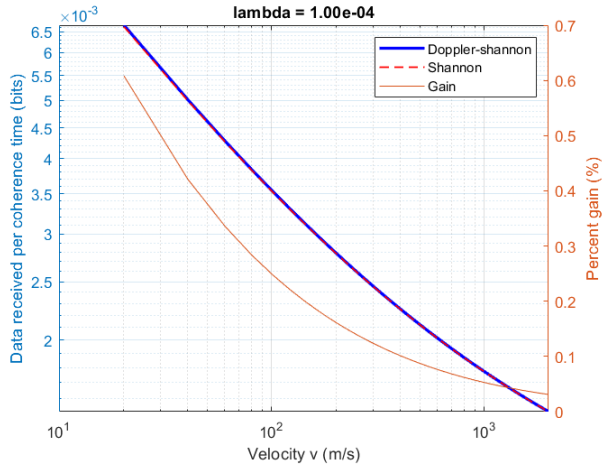
(b) $\lambda = 10^{-1}$



(c) $\lambda = 10^{-2}$

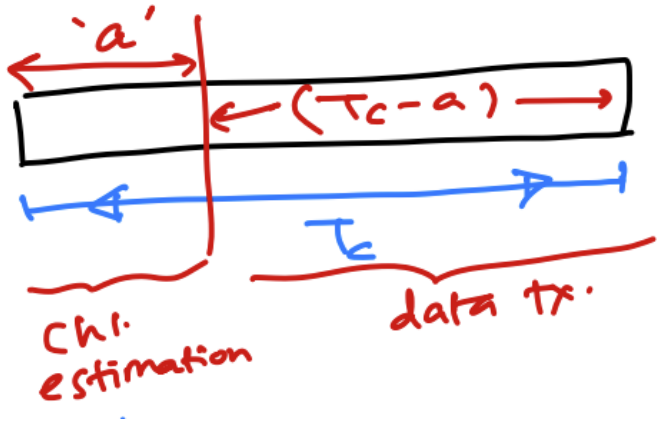


(d) $\lambda = 10^{-3}$



(e) $\lambda = 10^{-4}$

Doppler aware utilities - 2



- In time division multi-access systems (used in IoT V2X applications), the coherence time interval has a "channel estimation time" (a) followed by a "data transmission time" ($T_C - a$).
- Net data transmitted $= (T_{c_i} - a)B log_2(1 + K_i r_i^{-\alpha})$
- Data rate, $U_i = \left(1 - \frac{a}{Tc_i}\right)^+ B \log_2(1 + K_i r_i^{-\alpha})$ [bits per sec]
- Note: if $(a > Tc)$, U_i becomes negative [practically channel estimation cannot exceed the coherence time interval]
- For $(a << Tc)$ [very efficient channel estimation]: effect of Doppler vanishes (Doppler-Shannon = Shannon association)
- For $(a < Tc)$: effect of Doppler is visible on system performance

Further,

$$Tc > a \tag{29}$$

$$\frac{k_1 \lambda_w}{v|cos\theta|} > a \tag{30}$$

$$\frac{k_1 c}{v|cos\theta| f_c} > a \tag{31}$$

Hence,

$$U_i = \left(1 - \frac{av|cos\theta_i| f_c}{k_1 c}\right)^+ B \log_2(1 + K_i r_i^{-\alpha}) \tag{32}$$

Utility captures affect of velocity and carrier frequency too.

Doppler-Shannon UE association

Doppler-Shannon failure metric:

$$x^* = \arg\max_i U_i \tag{33}$$

$$\mathbb{P}(\max_i \left(1 - \frac{a}{Tc_i}\right)^+ B \log_2\left(1 + K_i r_i^{-\alpha}\right) < \tau) \tag{34}$$

$$= \exp\left(-\frac{\lambda}{2}\mathbb{E}[K^{2/\alpha}] \int_\theta \left(2^{\tau/\left(\left(1-\frac{a}{Tc(\theta)}\right)^+ \times B\right)} - 1\right)^{-2/\alpha} \mathrm{d}\theta\right) \tag{35}$$

Success metric:

$$\int_{\tau=0}^{\infty} \left(1 - \exp\left(-\frac{\lambda}{2}\mathbb{E}[K^{2/\alpha}] \int_\theta \left(2^{\tau/\left(\left(1-\frac{a}{Tc(\theta)}\right)^+ \times B\right)} - 1\right)^{-2/\alpha} \mathrm{d}\theta\right)\right) \mathrm{d}\tau \tag{36}$$

Classical Shannon based UE association

$$U_i = \left(1 - \frac{a}{Tc}\right)^+ B \log_2(1 + K_i r_i^{-\alpha}) \tag{37}$$

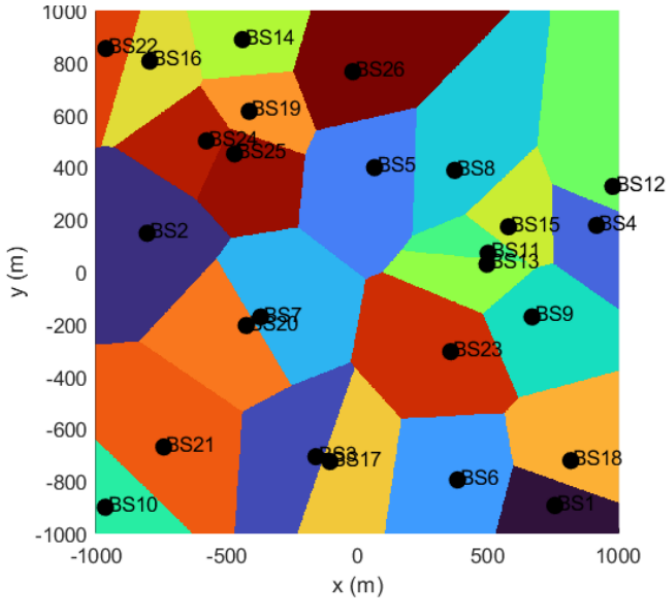
$$\mathbb{P}(\max_i \left(1 - \frac{a}{Tc}\right)^+ B \log_2 \left(1 + K_i r_i^{-\alpha}\right) < \tau) \tag{38}$$

$$= \exp\left(-\frac{\lambda}{2} 2\pi \ \mathbb{E}[K^{2/\alpha}] \left(2^{\tau/\left(\left(1-\frac{a}{Tc}\right)^+ \times B\right)} - 1\right)^{-2/\alpha}\right) \tag{39}$$

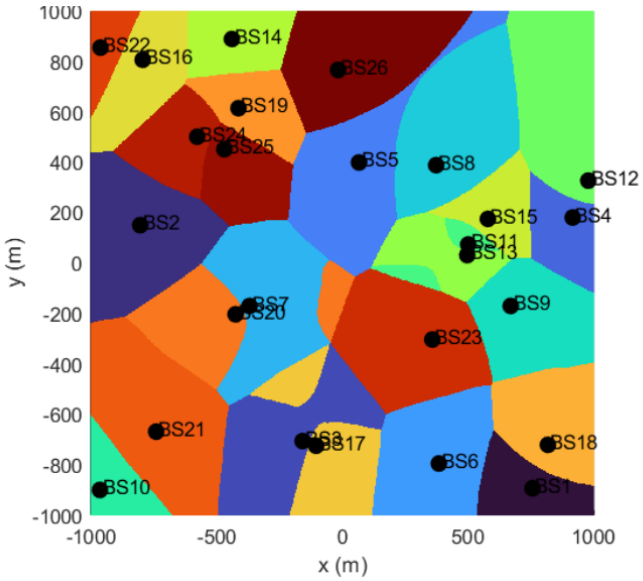
By Jensen's inequality, (Shannon $\geq$ Doppler-Shannon) [failure metric]

$$\begin{aligned} &\exp\left(-\frac{\lambda}{2} 2\pi \ \mathbb{E}[K^{2/\alpha}] \left(2^{\tau/\left(\left(1-\frac{a}{Tc}\right)^+ \times B\right)} - 1\right)^{-2/\alpha}\right) \geq \\ &\exp\left(-\frac{\lambda}{2} \mathbb{E}[K^{2/\alpha}] \int_\theta \left(2^{\tau/\left(\left(1-\frac{a}{Tc(\theta)}\right)^+ \times B\right)} - 1\right)^{-2/\alpha} \mathrm{d}\theta\right) \end{aligned} \tag{40}$$

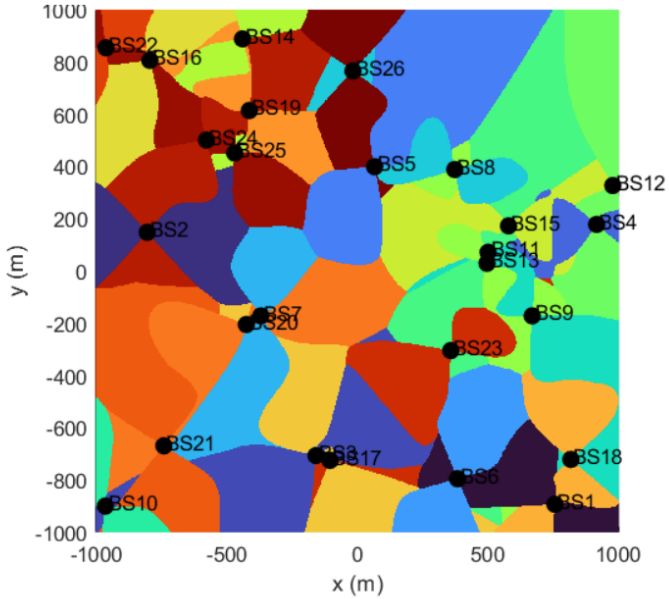
or, Shannon $\leq$ Doppler-Shannon [success metric]



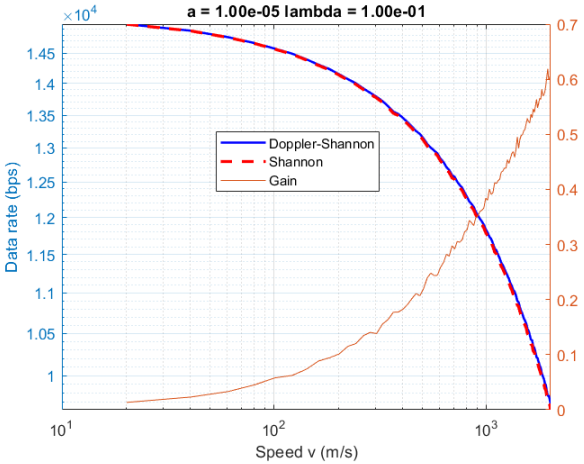
(a) $a = 10^{-5}$(very small)



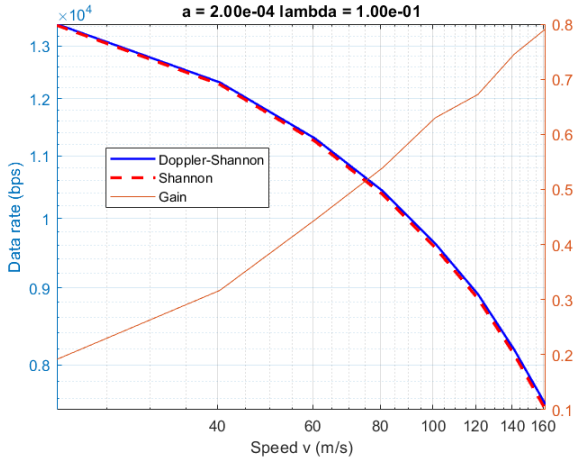
(b) $a = 10^{-3}$, $v = 100mps$



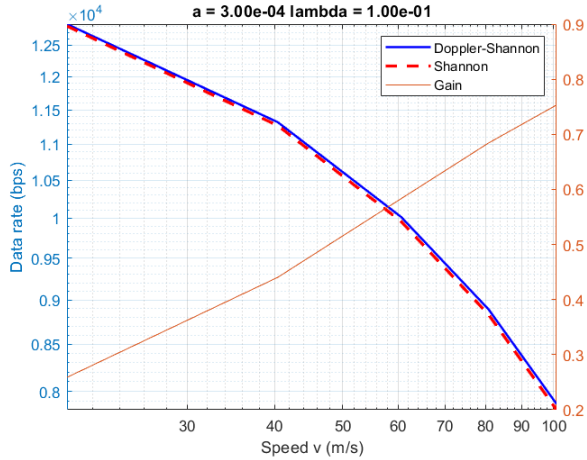
(c) $a = 10^{-3}$, $v = 200mps$



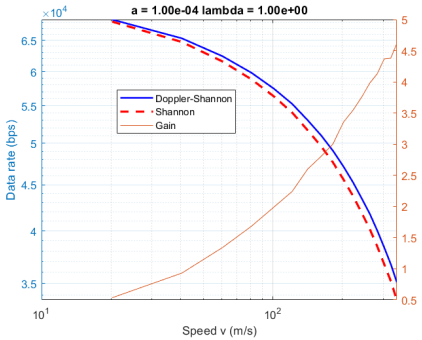
(a) $\lambda = 10^{-1}, a = 10^{-5}$



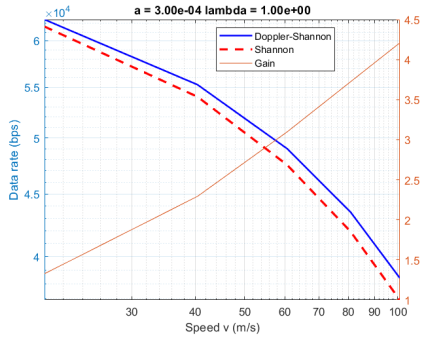
(b) $\lambda = 10^{-1}, a = 2 \times 10^{-4}$



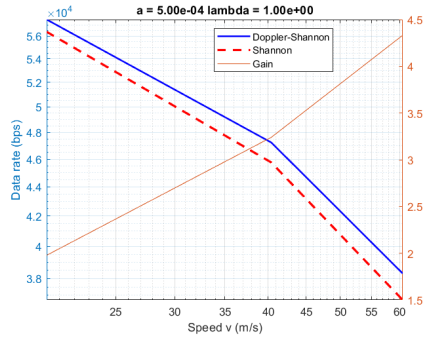
(c) $\lambda = 10^{-1}, a = 3 \times 10^{-4}$



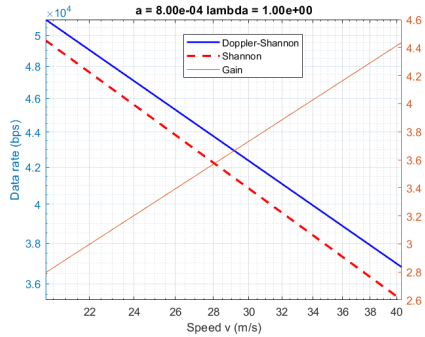
(d) $\lambda = 1, a = 10^{-4}$



(e) $\lambda = 1, a = 3 \times 10^{-4}$



(f) $\lambda = 1, a = 5 \times 10^{-4}$



(g) $\lambda = 1, a = 8 \times 10^{-4}$

Doppler aware utilites - 3

$$U_i = \frac{B}{Tc_i} log_2(1 + K_i r_i^{-\alpha}) \text{ [bits per sec per coherence time]} \quad (41)$$

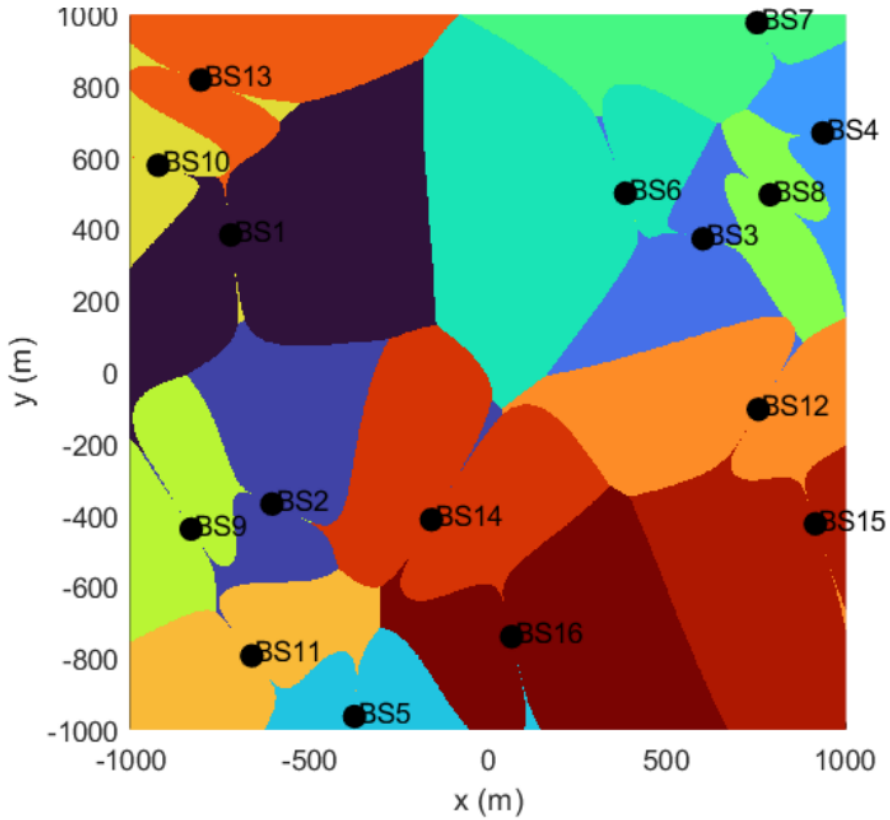
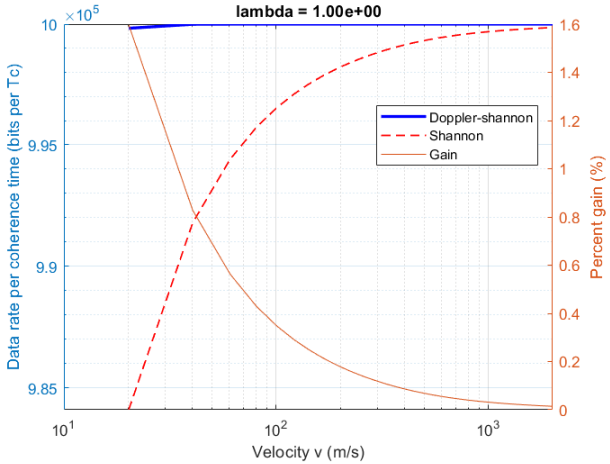
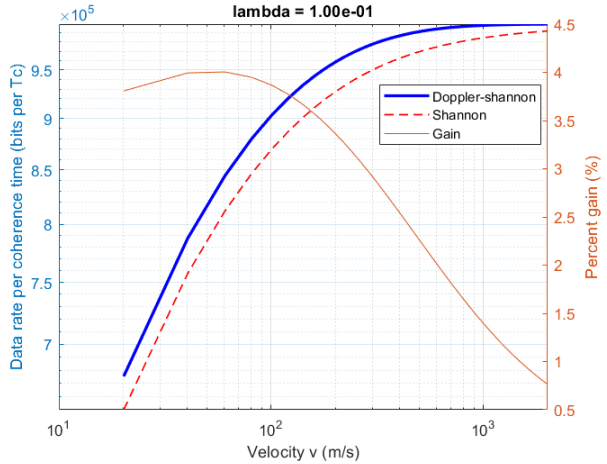


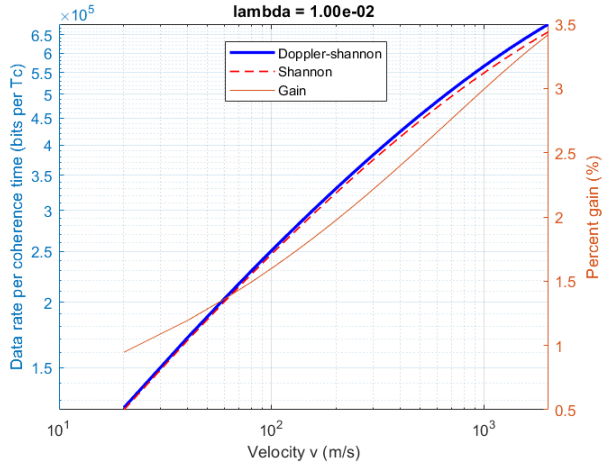
Figure 7: Caption



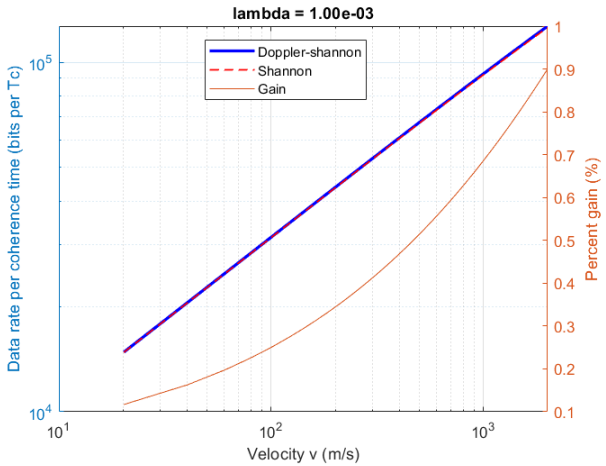
(a) $\lambda = 1$



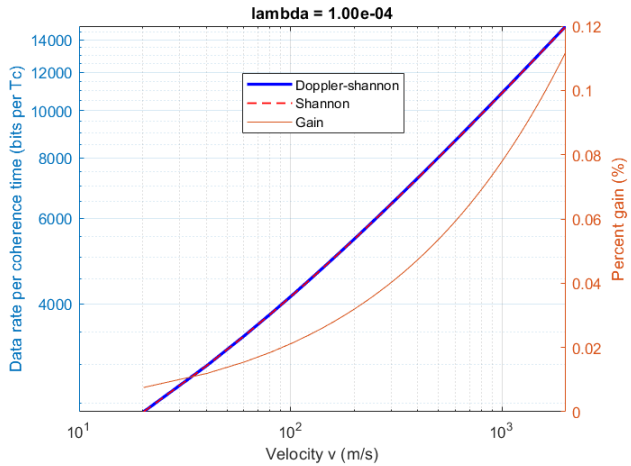
(b) $\lambda = 10^{-1}$



(c) $\lambda = 10^{-2}$



(d) $\lambda = 10^{-3}$



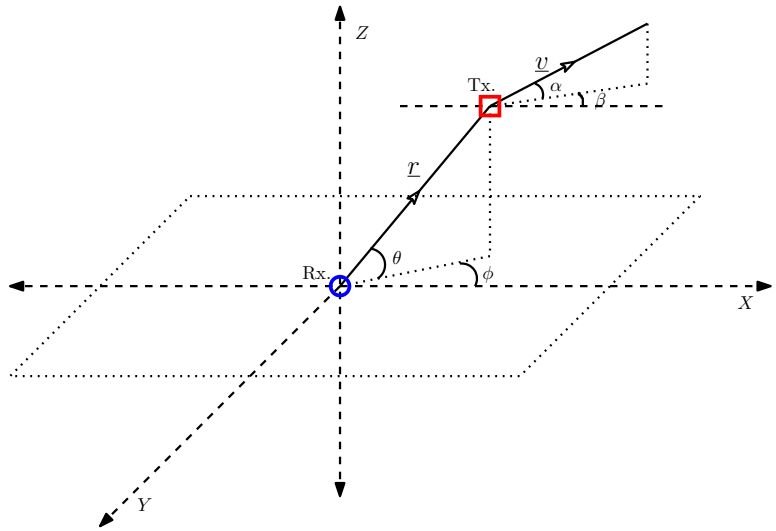
(e) $\lambda = 10^{-4}$

3D analysis

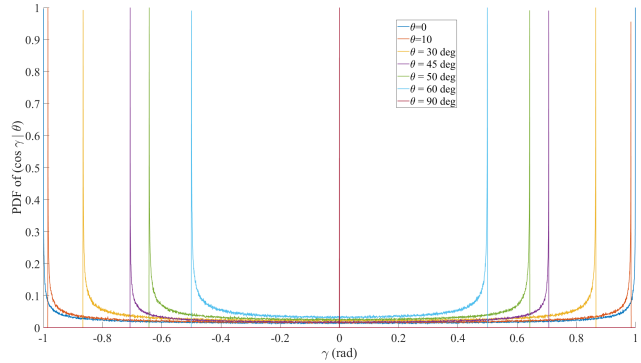
- Four angles (2 azimuth, 2 elevation)
- Elevation angles are non-uniformly distributed ($cos\theta/2$)
- Velocity component along LoS

$$cos\gamma = \cos\alpha\cos\theta\cos\phi + \sin\alpha\sin\theta \sim U[-1,1]$$
$$\sim \cos\theta\cos\phi \sim U[-1,1] \tag{42}$$

- Doppler is "uniformly distributed" in 3D; when conditioned on elevation gives the 2D arc-sine distribution.
- Failure metric [Doppler-Shannon]:
$exp\left(-\frac{\lambda}{3}\mathbb{E}[K^{3/\alpha}]\int_\theta\int_\phi(f(Tc(\theta,\phi)))^{-3/\alpha}\cos\theta d\theta d\phi\right)$



(a)



(b)

Other works

1. Investigation of RIS aided NTNs (with F. Baccelli, Junse Lee)
   - Study of line of sight visibility between UE-RIS, RIS-aerial BS, and UE-aerial BS
   - Study the joint visibility of a RIS, from UE as well as aerial BS.
   - SNR improvement observed through the geometry (to be done).
   - Manuscript under preparation
2. Study of Joint Communication and Sensing (JCAS) for large networks (with F. Baccelli, Nahuel S. Loto)
   - Initial 1D model developed, extending it to 2D
3. Study of routing in LEO satellite constellations (with P. Martins)
4. Timing advance estimation in LEO satellites using EKF (with P. Popineau, P. Martins) [1][1]

[1] A. Balakrishnan, P. Popineau, and P. Martins, "Timing advance and Doppler shift estimation in LEO satellite networks: A recursive Bayesian study," accepted in Proc. IEEE GLOBECOM, pp. 1-6, Taipei, Taiwan, Dec. 2025.