Dynamique et Géométrie des Réseaux de Constellations de Satellites

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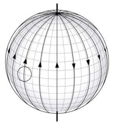
Retraite du LINCS

Structure of the presentation

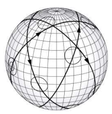
- **1. LEO/MEO Satellite Constellations**
  - **Walker classification**
  - **Functionalities**
- **2. NTN based on LEO Satellite Constellations**
- **3. Engineering Challenges**
- **4. Geometry**
- **5. Dynamics**
- **6. National and International Actions**

1. LEO/MEO SATELLITE CONSTELLATIONS

- **Walker's classification** - simplest version
  - Circular orbits
  - All altitudes equal
  - Periodic arrangement of orbits
  - Periodic arrangement of satellites



Walker star

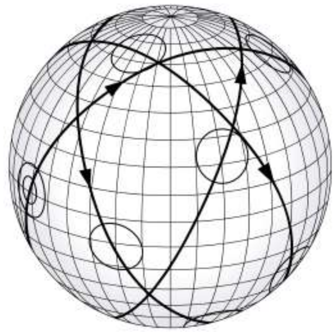


Walker delta

- **Lots of variants**, e.g., multi-altitudes

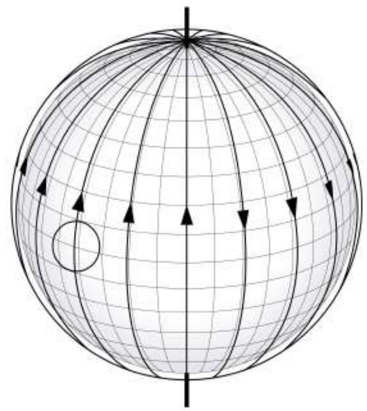
LEO: Low Earth Orbit, MEO: Medium Earth Orbit constellations

Periodic arrangement of orbits



Walker Delta
basis of Starlink and Galileo

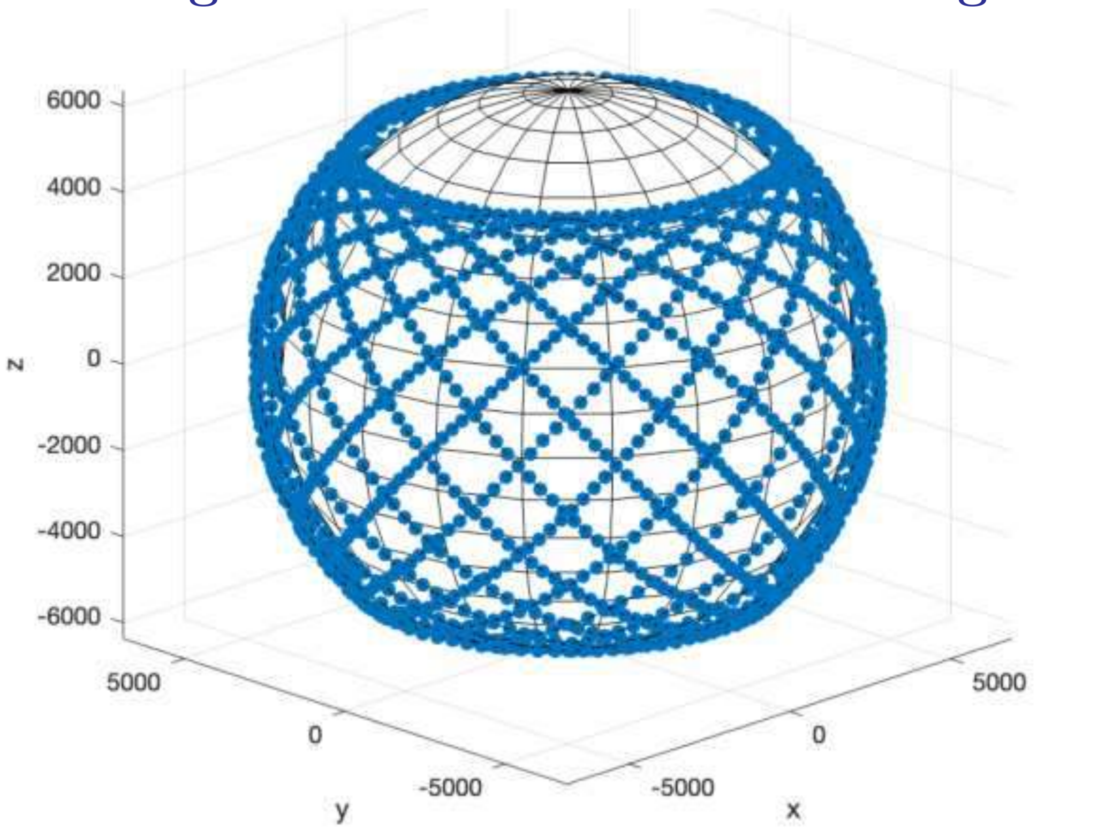
- Orbital planes: all same inclination
- Ascending points: period. on 360^o of Equator



Walker Star
basis of Oneweb

- Inclinations of orbits: $\sim 90^o$ (quasi polar)
- Orbital planes: periodic on 180^o of equator

Periodic arrangement of satellites on each given orbit



Walker Delta

FUNCTIONALITIES of CONSTELLATIONS

– **Communications**
few constellations, each with a large number of satellites

– **Earth observation**
many constellations, each with fewer satellites

– **Geolocalisation**
few constellations, each with few satellites

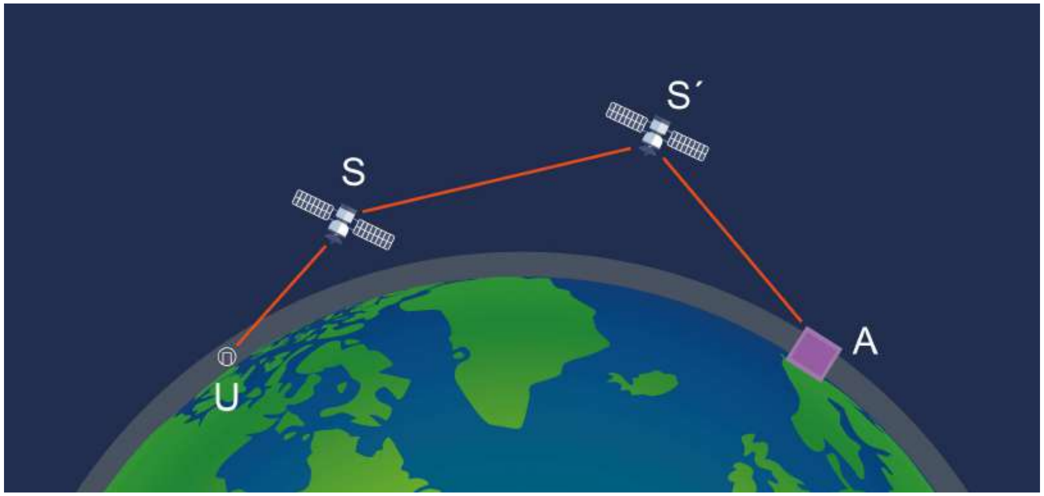
Structure of the presentation

- 1. LEO/MEO Satellite Constellations
- 2. NTN based on LEO Satellite Constellations
  – Broadband functionalities
  – Instances of economic models
  – Sovereignty issues
- 3. Engineering Challenges
- 4. Our System Level Analysis
- 5. National and International Actions

2. SATELLITE CONSTELLATIONS FOR INTERNET ACCESS

- **Coverage solution which is**
  - **universal (latitude constraints for some configurations)**
  - **without interruptions (if many satellites)**
  - **broadband (constraints on number of users per satellite)**
  - **low latency (because low orbits)**
  - **resilient (distributed system)**

COMMUNICATIONS BETWEEN SATELLITES



Credits hal-04626677v1

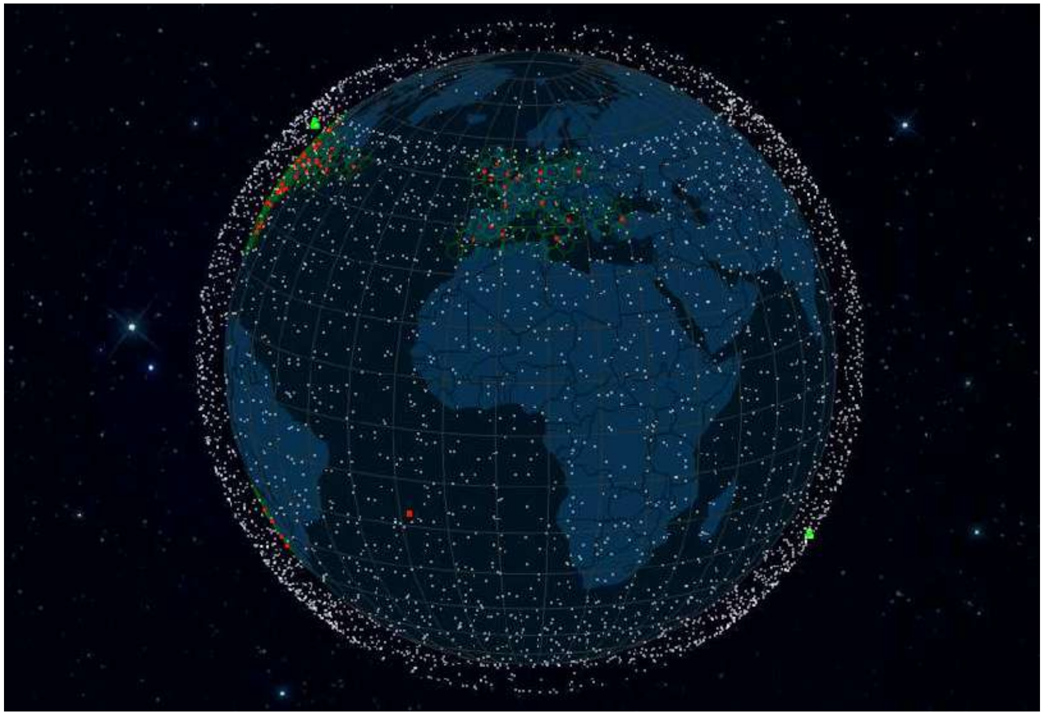
U: *UE*, **S:** *satellite*, **A:** *gateway station*

Examples of S-U and S-A links:

Ku (10,7-14,5 GHz), Ka (17,3-30 GHz), V (37-50,4 GHz) & 5G Bands

Inter satellite links: *Laser*

THE STARLINK CONSTELLATION



8K satellites end of 2024, authorization for 42 K
Altitude : between 350 and 550 km

Structure of the presentation

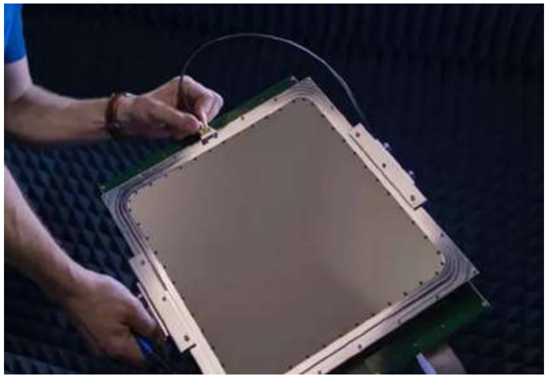
- 1. LEO/MEO Satellite Constellations
- 2. NTN based on LEO Satellite Constellations
- 3. Engineering Challenges
  - Physical layer
  - Coverage and Spectral Efficiency
  - BS Mobility
- 4. Our System Level Analysis
- 5. National and International Actions

3. ENGINEERING CHALLENGES

- **Spectral Efficiency**
  - Coverage
  - Spectral efficiency
  - TN–NTN interference
  - Antennas, RIS enhanced
- **Mobile Base Stations**
  - Doppler Shift
  - Beam management
  - Handover
  - Multi-hop routing
- **TN-NTN Interplay**
  - 5G offloading
  - Vertical handovers
- **NT Edge Computing**
  - Regenerative Satellites
  - Combination of
    - observation,
    - computing and
    - communication
  - Spatial Internet

WHY ARE THESE CHALLENGES? : PHYSICAL LAYER

- **Far Away BSs**
  - **Low power signal**
  - **Need for new UE antennas**
  - **Need for new waveforms**

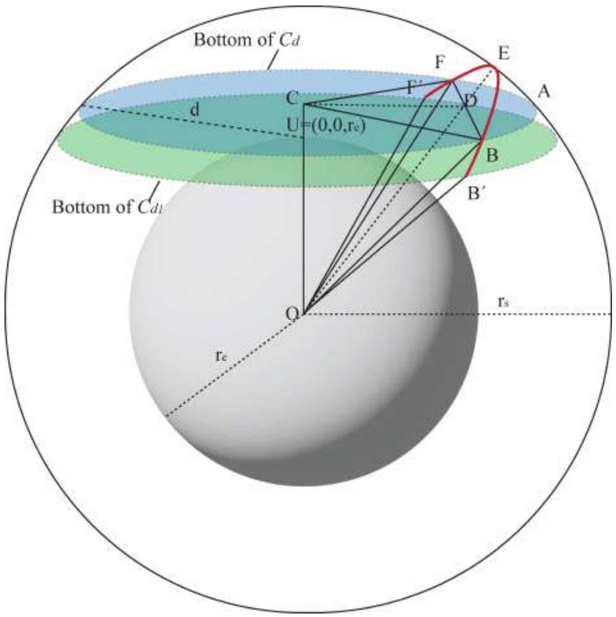


Greenerwave's RIS based UE Antenna

Further RIS based solutions to be discussed later

WHY ARE THESE CHALLENGES? : SPECTRAL EFFICIENCY

- **Spherical Geometry**
  - New coverage properties
  - New visibility cap constraints
  - New def. of association cells
  - New structure of interference
  - New spectral efficiency



Visibility Cap
Visibility Arc of an Orbit

WHY ARE THESE CHALLENGES? : BS MOBILITY

- **Dynamical BSs Moving on Orbits**
  - **Lower coherence times (Doppler)**
  - **Higher handovers frequency**
  - **New multihop routing mech.**
  - **New types of handovers**

Dynamic Shortest Path Routing
Credits M. Hanley, UCL

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- **3. Engineering Challenges**
- **4. Geometry**
  - **Walker full deployment**
  - **Partial deployment: Cox, Poisson**
- **5. Dynamics**
- **6. National and International Actions**

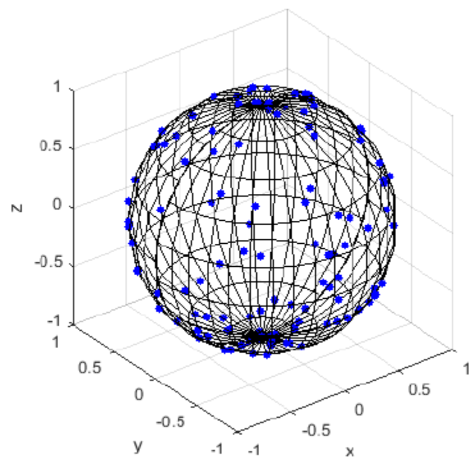
4. GEOMETRY OF NTN CELLULAR NETWORKS

- **Need for new system level analysis tools**
- **System level questions How does orbital + satellite organization translates into statistics for**
  - **Coverage**
  - **Statistics of Spectral Efficiency**
  - **Interaction/Interference between 5G and NTN**
  - **5G offloading Capacity**

GEOMETRY: MAIN RESULTS

- **Spatial Averages :**
  - **Shortest sat connection, route GP1**
  - **No visible satellite probability**
  - **Mean number of visible satellites**
  - **Coverage probability**
  - **Distribution of downlink SINR and Shannon rate GP2**

GEOMETRY: INITIAL STEPS



An Isotropic Binomial PP on the sphere
[Okati et al. 20] IEEE Trans. Comm.

Binomial p.p. with satellites uniformly distributed on a sphere
Basic questions similar to those on the plane but in spherical geometry
Limitations

1. **Geometry: No orbital planes**
2. **Analysis: clustering of interference ignored**

GEOMETRY: NEED FOR MORE ACCURATE MODELS

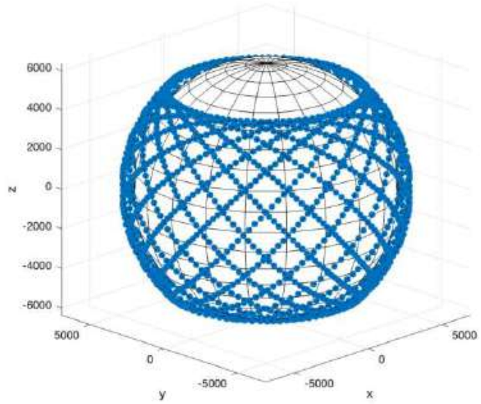


Figure: Starlink constellation

Starlink	Geometry	Binomial
Yes	orbits	No
Yes	clustering	No

Goal: a new model for satellites on orbits

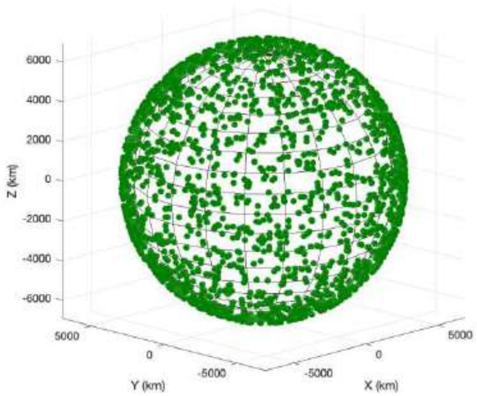
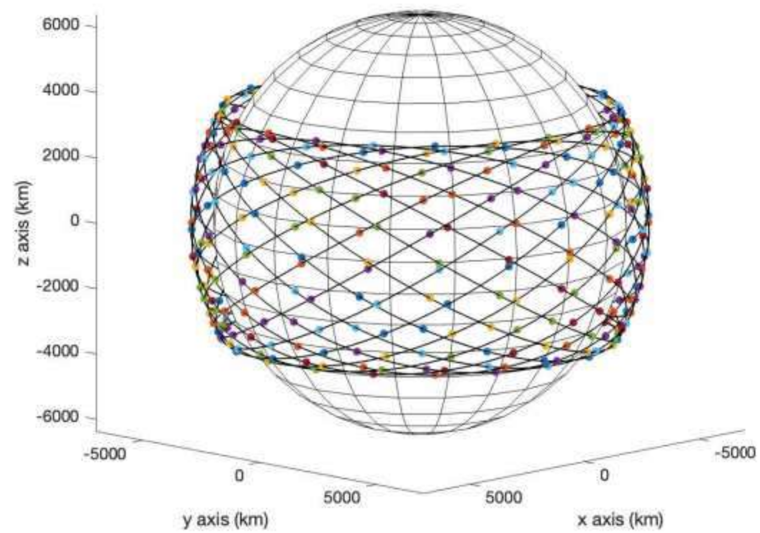


Figure: binomial model

GEOMETRY: WALKER

"Stationary" stochastic geometry framework for Walker Constellations

- Orbital planes with fixed inclination, regularly spaced longitudes and random phase
- Satellites equally spaced on orbits, with random phase
- Evaluation of SINR distribution and spectral efficiency



[C.S. Choi, F.B., Tr. Veh. Tech. 25]

WALKER DELTA: MODEL/NOTATION

- $\mathbf{N_o}$ number of orbits
- The inclinations of orbits are all equal to ϕ
- The $\mathbf{N_o}$ ascending points are distributed regularly on the equator $[\mathbf{0}, \mathbf{2}\pi]$ with initial phase phase $\bar{\theta} \sim \mathbf{Uniform}(0, \frac{2\pi}{N_o})$
- $\mathbf{N_s}$ number of satellites on each orbit
- Phases of the N_s satellites on each orbit: periodic on $[\mathbf{0}, \mathbf{2}\pi]$
- The initial phase of the j-th satellite on the i-th orbit is

$$\omega_\mathbf{j} = \mathbf{2}\pi\mathbf{j}/\mathbf{N_s} + \bar{\omega} \mod \mathbf{2}\pi,\ \mathbf{j} = \mathbf{1}, \ldots, \mathbf{N_s} \text{ with } \bar{\omega} \sim \mathbf{Uniform}(\mathbf{0}, \frac{\mathbf{2}\pi}{\mathbf{N_s}})$$

- $\mathbf{X_{i,j}}$: location of satellite of phase $\omega_\mathbf{j}$ on orbit i
- Constellation P.P. at $\mathbf{t} = \mathbf{0}$: $\mathbf{\Psi} = \sum_{\mathbf{i=1}}^{\mathbf{N_o}} \sum_{\mathbf{j=1}}^{\mathbf{N_s}} \delta_{\mathbf{X_{i,j}}}$

GEOMETRY: GP1: STATISTICS OF DISTANCE TO NEAREST SATELLITE

- **Let $D(l_u)$ be the distance from the typical user at latitude l_u to its nearest satellite. $D(l_u) = \infty$ if no satellite is visible from the typical user**

- **Theorem [C.S. Choi, F. B., Tr. Veh. Tech. 25]**
  For the Walker Delta, for $r - e < d < \sqrt{r^2 - e^2}$,

$$P(D(l_u) > d) = \kappa \int_0^{\frac{2\pi}{N_o}} \int_0^{\frac{2\pi}{N_s}} 1 \left\{ \frac{r^2 + e^2 - d^2}{2re} > \max \frac{X_{i,j} \cdot u}{\|X_{i,j}\| \|u\|} \right\} d\omega \, d\theta,$$

with $\kappa = N_o N_s / (4\pi^2)$

GEOMETRY: GP1: STATISTICS OF DISTANCE TO NEAREST SATELLITE (*continued*)

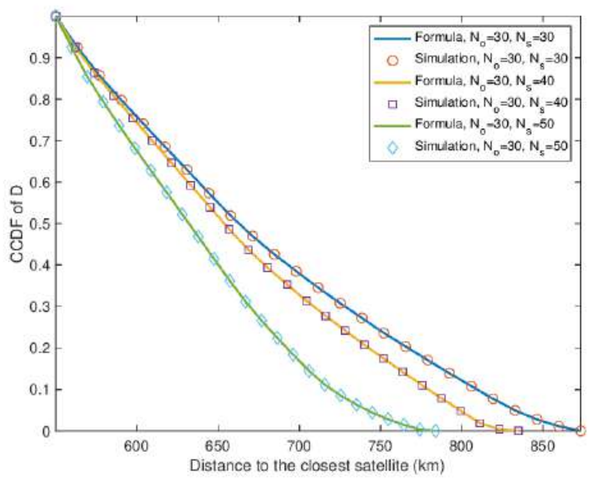


Fig. 2. The CCDF of random variable D_l. Here, $r = 6921$ km. The system-level simulation results confirm the accuracy of the derived formula.

- **Thanks to the deterministic structure of the deployment of satellites, for latitudes l_u smaller than a threshold L, for N_o and N_s large enough, there exists a constant $d_c(l_u) \leq \sqrt{r^2 - e^2}$ such that $P[D(l_u) \leq d] = 1$ for all $d > d_c(l_u)$**

For instance when $N_o = 30$ and $N_s = 50$, we have $P(D(l_u) > d_c(l_u)) = 0$ for $d_c(l_u) \approxeq 775$km, i.e., the association distance is for sure less than 775 km, whereas the max due to visibility is 2108 km

GEOMETRY: GP2: LAPLACE TRANSFORM OF INTERFERENCE

- **Theorem** [C.S. Choi, F. B., Tr. Veh. Tech. 25]
  In the Walker delta constellation, the Laplace transform of the total interference of the typical user at u only depends on its latitude and is given by

$$\kappa \int_0^{\frac{2\pi}{N_o}} \int_0^{\frac{2\pi}{N_s}} \mathbf{e}^{\left(\sum c_{i,j} \log\left(\mathcal{L}_H\left(\frac{spG_{i,j}}{\|X_{i,j}-u\|^\alpha}\right)\right)\right)} \, d\bar{\omega}\, d\bar{\theta},$$

where $c_{i,j} = 1\left\{\frac{X_{i,j}\cdot u}{\|X_{i,j}\|\|u\|} \geq \frac{e}{r}\right\}$ **and**

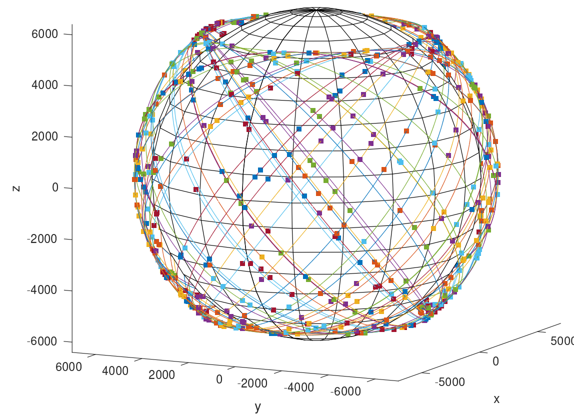
$$G_{i,j} = \begin{cases} g_t g_r & \text{if } \frac{X_{i,j}\cdot u}{\|X_{i,j}\|\|u\|} \geq \frac{r^2+e^2-d_g^2}{2re}, \\ g_r & \text{if } \frac{X_{i,j}\cdot u}{\|X_{i,j}\|\|u\|} < \frac{r^2+e^2-d_g^2}{2re}. \end{cases}$$

- **More results on distribution downlink Shannon rate at a given latitude**

GP2: CASE OF PARTIAL DEPLOYMENT OF WALKER

Cox-type stochastic geometry framework with

- **Poisson orbits with fixed inclinations and random ascending points**
- **Poisson satellites on orbits**
- **Evaluation of SINR distribution and spectral efficiency**



[C.S. Choi, F. B., IEEE Tr. Com. 25]

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- **5. Dynamics**
  - **Ephemerides, routing : Walker case**
  - **Handover frequency : Poisson case**
- **6. National and International Actions**

DYNAMICS: MAIN RESULTS ON EPHEMERIDES

- **Invariant measures DP1**
- **Ergodicity DP2**
- **Time Averages**

$\times$

- **Shortest Sat connection, shortest route**
- **Number of visible Sats**
- **SINR and Shannon rate**

5. DYNAMICS: THE WALKER CONSTELLATION POINT PROCESS AS A DYNAMICAL SYSTEM

- $\mathbf{x} = (\bar{\omega}, \bar{\theta})$: initial phases on the torus $[0, \frac{2\pi}{N_o}] \times [0, \frac{2\pi}{N_s}]$
- $\Psi(x, t)$: Walker satellite point process at time t for initial x
  Full description of the ephemerides of the constellation
- $\Psi(x, t)$ obtained from $\Psi(x, 0)$ by a torus translation $R(x, t)$
- $\Psi(x, t)$ is a deterministic measurable functional Ξ of $R(x, t)$
- Therefore, if R has an invariant probability measure $\mathcal{P}$ on S, then Ψ has an invariant probability measure as well, which is the push forward of $\mathcal{P}$ by Ξ, denoted by $\mathcal{P}_\Psi$
- If R is ergodic w.r.t. $\mathcal{P}$, so is Ψ w.r.t. $\mathcal{P}_\Psi$

WALKER DYNAMICS: DP1: INVARIANT PROB. MEASURE

- **Let $\overline{\mathcal{Q}}$ denote the product of the two uniform distributions on $\mathbf{S} = [\mathbf{0}, \frac{2\pi}{\mathbf{N_o}}] \times [\mathbf{0}, \frac{2\pi}{\mathbf{N_o}}]$**
- **Let $\overline{\mathcal{Q}}_{\mathbf{\Psi}}$ denote the distribution of Ψ at time zero when $(\bar{\theta}, \bar{\omega})$ has distribution $\overline{\mathcal{Q}}$**
- **Theorem [C.S. Choi, F. B., IEEE Veh. Tech. 25]**
  The probability measure $\overline{\mathcal{Q}}$, on S, is invariant for R and the probability measure $\overline{\mathcal{Q}}_{\mathbf{\Psi}}$, on $\mathcal{N}_{\mathbf{r}}$, is invariant for $\mathbf{\Psi}$

WALKER DYNAMICS: DP2: ERGODICITY, PERIODICITY

- **Rotation speed ratio:**

$$\rho = \frac{\bar{\mathbf{v}}_\theta}{\bar{\mathbf{v}}_\omega}$$

- **Theorem [C.S. Choi, F. B., IEEE Veh. Tech. 25]**

  - **If ρ is irrational, then the dynamical system**
    - **R on S is minimal and ergodic w.r.t. $\overline{\mathcal{Q}}$**
    - **Ψ on $\mathcal{N}_r$ is ergodic w.r.t. $\overline{\mathcal{Q}}_\Psi$**
  - **If the ratio ρ is rational, then both R and Ψ are periodic**

WALKER DYNAMICS: DP2: ERGODICITY, PERIODICITY (*continued*)

- **When ρ is irrational, for $\overline{\mathcal{Q}}$ almost all $\mathbf{x}$**

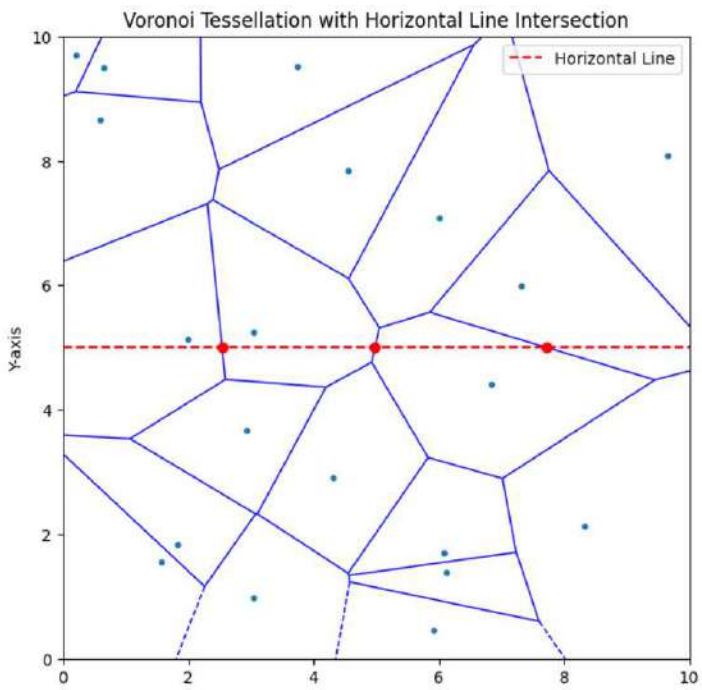
$$\lim_{\mathbf{T}\to\infty} \frac{\mathbf{1}}{\mathbf{T}} \int_{\mathbf{0}}^{\mathbf{T}} \mathbf{h}(\mathbf{\Psi}(\mathbf{x},\mathbf{t}))\mathrm{d}\mathbf{t} = \int \mathbf{h}(\psi)\overline{\mathcal{Q}}_{\mathbf{\Psi}}(\mathrm{d}\psi),$$

for all functions $\mathbf{h} : \mathcal{N} \to \mathbb{R}$ such that $\int |\mathbf{h}(\psi|)\overline{\mathcal{Q}}_{\mathbf{\Psi}}(\mathrm{d}\psi) < \infty$

- **When ρ is rational, the system is periodic. The long time average (*e.g.*, associated with the ephemerides) of a given Earth user does not match averages w.r.t. the invariant distribution $\overline{\mathcal{Q}}_{\mathbf{\Psi}}$**

- **Altitude determines the satellite speed**

HANDOVER DYNAMICS 1 (HISTORY) : STATIC STATIONS AND MOBILE USER



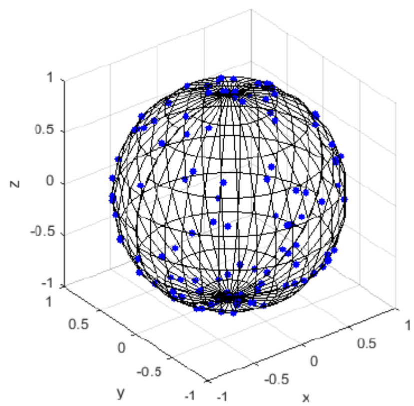
Trajectory of a mobile user along a straight line, among static stations

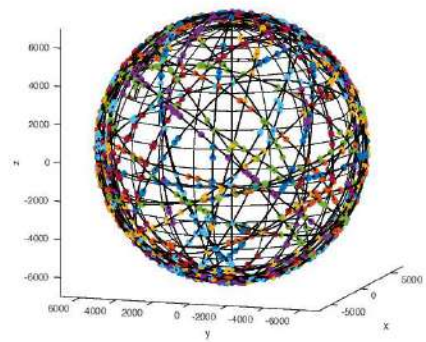
Handover frequency: static case

Theorem[B. & Zuyev, 1997] Suppose the user is moving in a straight line with speed v and base stations are static Poisson with intensity λ. Then the handover frequency is

$$\lambda_h = \frac{4v\sqrt{\lambda}}{\pi}$$

HANDOVER DYNAMICS 2: MOBILE STATIONS AND STATIC USER





An isotropic Poisson PP of stations with isotropic motions on the sphere and a Cox point process of stations moving on isotropic Poisson orbits

– **Dynamical versions** of the **static versions** studied above

– **→ Local planar "projection" seen from a tagged Earth user**

DYNAMICS 2: POISSON PLANAR "PROJECTION"

- **Initial station locations: Φ, a PPP of intensity λ on $\mathbb{R}^2$**
- **i.i.d. motion directions : θ_i, e.g. Uniform$[-\pi, \pi)$, $\forall \mathbf{X_i}$ in Φ**
  Results hold for all distributions of motion direction
- **Velocity vector: $\mathbf{V_i} = \mathbf{v}(\cos\theta_\mathbf{i}, \sin\theta_\mathbf{i})$, where v constant**
- **Locations at time t: $\mathbf{X_i^t} := \mathbf{X_i} + \mathbf{V_i}.\mathbf{t}$**

$$\Phi^t := \{X_i^t : X_i \in \Phi\}$$

Static user at the origin

Theorem[Displacement theorem for PPP] For any time t, the point process Φ^t is homogeneous Poisson

DISTANCE FUNCTION, BIRD RANDOM CLOSED SETS

– **Let** $X \in \Phi$ **with:**

– **position** $\mathbf{X} = (|\mathbf{X}| \cos\psi, |\mathbf{X}| \sin\psi)$

– **Define** $\alpha = \psi - \theta$ **mod** 2π

– **velocity vector** $\mathbf{V} = \mathbf{v}(\cos\theta, \sin\theta)$

– **Distance function (of t):**

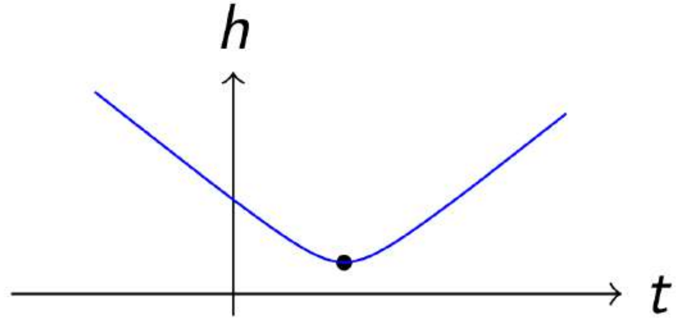
$$|X^t| = \sqrt{|X|^2 + 2tv|X|\cos\alpha + v^2t^2} := f_v(X,t)$$

– **Minimum of** $f_v(X,t)$, $(\mathbf{T_X}, \mathbf{H_X})$ **(head point):**

$$\left(-\frac{|X|}{v}\cos\alpha, |X||\sin\alpha|\right)$$

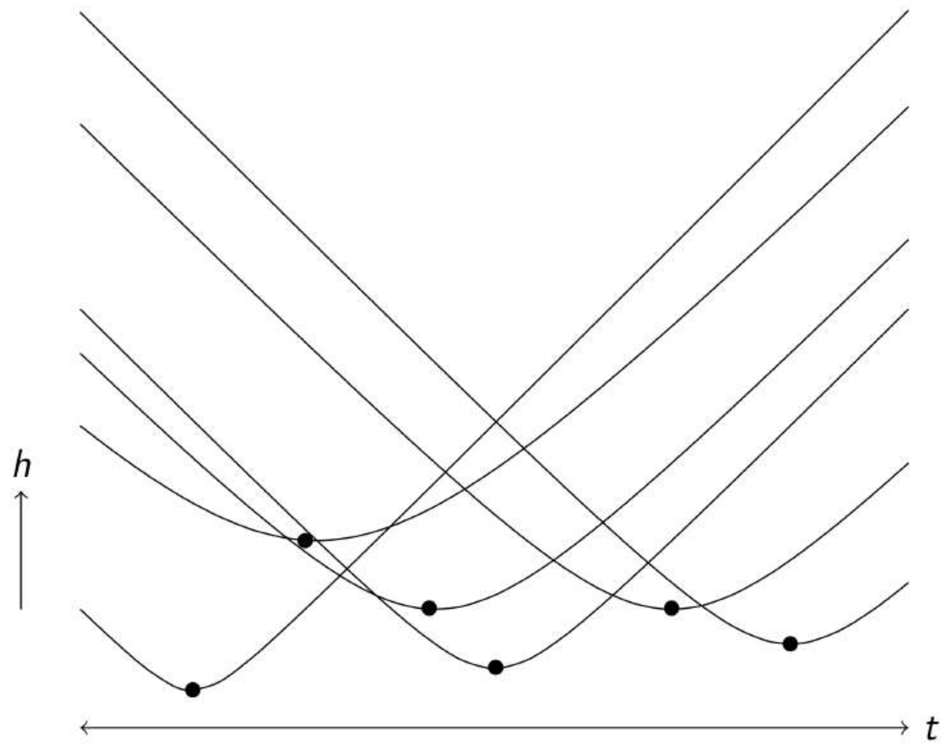
$\forall X \in \Phi \rightarrow$ **bird closed set:**

$$C_X := \{(t, f_v(X,t)) : t \in \mathbb{R}\}$$



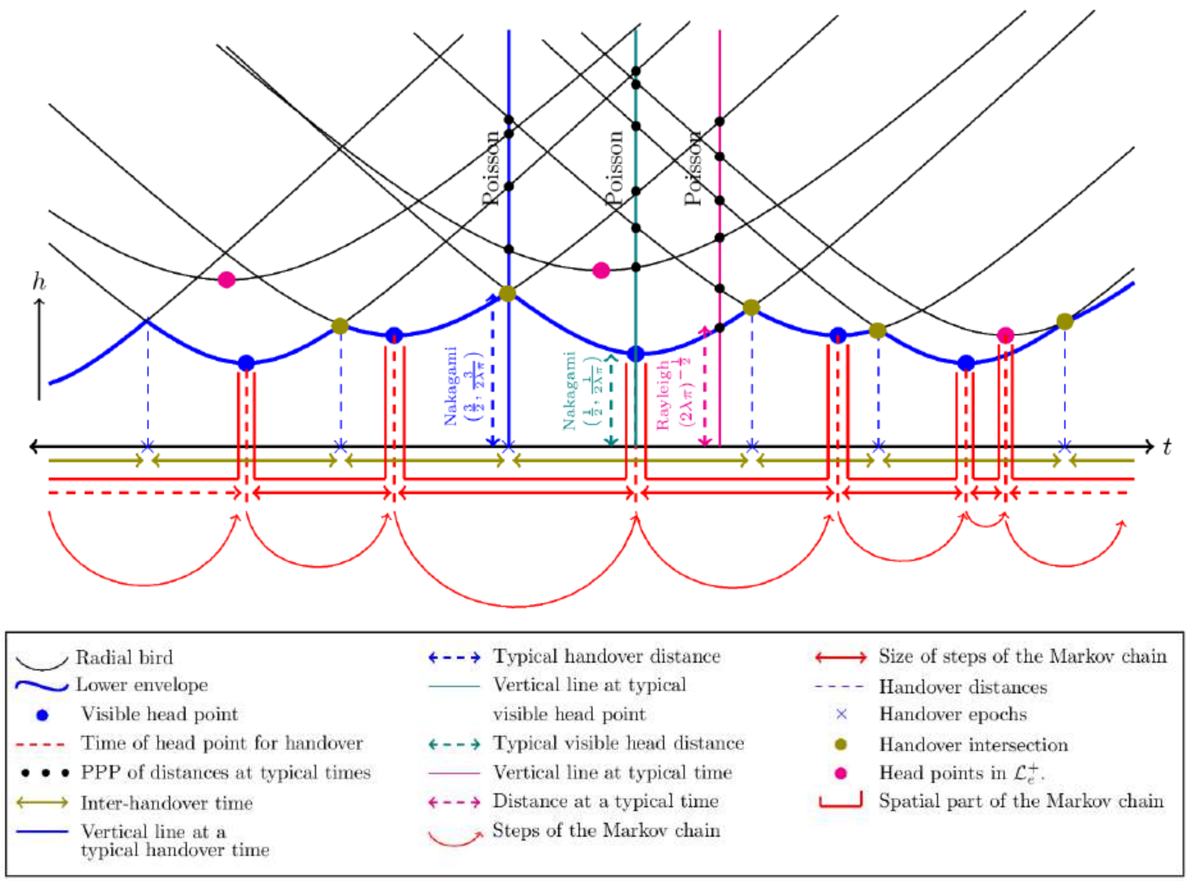
Branch of a hyperbola

THE BIRD PARTICLE PROCESS



The union of all bird closed sets forms the bird particle process
The head point process $\mathcal{H}$ is Poisson homogeneous on $\mathbb{H}^+ = \mathbb{R} \times \mathbb{R}^+$ with intensity measure $v dt \otimes 2\lambda \, dh$

OBJECTS OF INTEREST, SINGLE SPEED CASE



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- 6. National and International Actions
  - Ongoing research, Cooperations
  - Bibliography

5. ONGOING RESEARCH DIRECTIONS

- **Interplay between TN and NTN**
  (with Namyoon Lee, Korea University, SK)
  Hubert Curien

- **Frequency of Handovers: spherical, Walker**
  (with Sanjoy Kumar Jhawar/Emanuele Mengoli TP)
  ERC → BPI Univity/PEPR

- **Distribution of Doppler shift, Doppler based association**
  Ashutosh Balakrishnan, TP
  BPI Univity→ TP
  Pierre Popineau, CT
  ERC → Univity
  and Philippe Martins, TP

5. ONGOING RESEARCH DIRECTIONS (*continued*)

- **Distribution of SINR at Handovers**
  (with Sanjoy Kumar Jhawar, TP)
  ERC → BPI Univity

- **Routing**
  (with Sanjoy Kumar Jhawar/Emanuele Mengoli TP)
  ERC → BPI/PEPR
  (with Chang Sik Choi, Kaist)
  Hubert Curien

- **RIS enhanced NTNs**
  (with Junse Lee, Sungshin University, SK) → FINAL ZOOM
  Hubert Curien

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