Coverage and Capacity of Joint Communication and Sensing in Wireless Networks

INRIA Paris, LINCS

Romain Cervera

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Introduction

The article

- Title : *Coverage and Capacity of Joint Communication and Sensing in Wireless Networks*
- Date : 2022
- Authors : Nicholas R. Olson, Jeffrey G. Andrews, Robert W. Health, Jr.
- Main objective : Developing a conceptual framework for JCAS networks using an information theoretic formulation of sensing and communication coverage and capacity

Introduction

Joint Communication and Sensing (JCAS)

- Definition : networks combining communication functions with sensing services, like user localization or environmental sensing
- Examples : enable precision navigation in urban environment, monitor activity in a given coverage area, provide collision avoidance services to autonomous vehicles, facilitate AR/VR applications...
- Challenge : Dealing with the intercell interference with tractable models that can be simulated numerically

Introduction

Contributions and objectives

In constructing a modeling framework for JCAS network, here are two fundamental questions:

1. What is an appropriate conceptual framework for JCAS networks?
2. How can the performance of communication and sensing be characterized within this conceptual framework?

Until there, Stochastic Geometry has proven to be very powerful for SINR analysis in wireless communications. The main contribution of this article consists in extending that to radar tracking in JCAS networks.

Introduction

Outline of the article

1. Model of JCAS network and assumptions on it
2. An information theoretic approach for performance metrics
3. Models and assumptions for commmunication and sensing
4. Analytical expressions and approximations
5. Numerical results

Model of JCAS network and assumptions on it

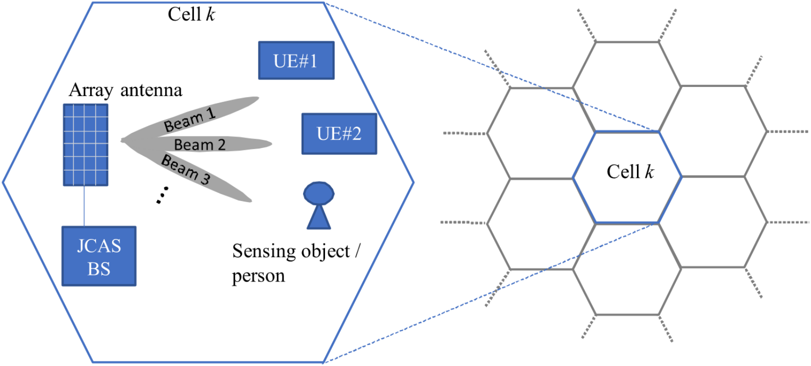


Figure: Basic scenario illustration[1] for JCAS in cellular systems

[1] Illustration from *Joint Design of Communication and Sensing for Beyond 5G and 6G Systems*, H. Wild, V. Braun, H. Viswanathan, IEEE

Model of JCAS network and assumptions on it

JCAS network representation

In general, the spatial components of a JCAS network may be represented by the tuple :

$$\{\Phi_B, \Phi_U, \Phi_S, \Theta_{\text{block}}\}$$

Where Φ_B is a PP on $\mathbb{R}^2$ modeling the locations of BSs, Φ_U is a PP on $\mathbb{R}^2$ modeling the locations of UEs, and Φ_S is a PP on $\mathbb{R}^2$ modeling the locations of SOs.
Θ_{block} is a set process on $\mathbb{R}^2$ representing the locations and shapes of blockages in the network.

Model of JCAS network and assumptions on it

Assumptions on spatial attributes

- Φ_B, Φ_U, Φ_S are stationary, mutually independent PPPs with $\lambda_U \ggg \lambda_B$ and $\lambda_S \ggg \lambda_B$.
- Θ_{block} is a Boolean line process whose induced LoS regions are approximated by the independent exponential blockage model.
- Communication may occur over either LoS and NLoS links, but sensing is LoS only.

An information theoretic approach for performance metrics

Communication capacity

For a link with $\mathrm{SINR}_{\mathrm{com}}$, the capacity over an AWGN[1] channel with interference treated as noise is given by the Shannon Formula. If one assumes that sinc-like pulses are employed, letting T_S denote the symbol rate of the system, we have:

$$C_{com} = \frac{1}{T_S} log_2(1 + \mathrm{SINR}_{\mathrm{com}})$$

[1] Additive (added to any noise that might be intrinsic to the information system) White (uniform power spectral density across the frequency band) Gaussian (normal distribution in the time domain) Noise

An information theoretic approach for performance metrics

Sensing capacity

For the radar, the expression of the capacity is more difficult to characterize, and then it is approximated[1] by the following lower bound:

$$C_{rad} \approx \frac{I(\boldsymbol{\theta} + \boldsymbol{N}_{est}; \boldsymbol{\theta})}{T_{CPI}} \approx \frac{1}{2T_{CPI}} log_2(|\boldsymbol{I} + \boldsymbol{Q}^{1/2}\boldsymbol{R}^{-1}\boldsymbol{Q}^{1/2}|)$$

Where $\boldsymbol{\theta}$ is the vector of unknown target parameters (range and velocity), $I(.;.)$ denotes mutual information and T_{CPI} is the processing interval. Here we assumed that the prior $P = \mathcal{N}(\bar{\boldsymbol{\theta}}, \boldsymbol{Q})$ is Gaussian and $N_{est} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{R})$ (vector of estimation noise).

[1] This estimation is inspired from *Extending Joint Radar-Communications Bounds for FMCW Radar with Doppler Estimation*, B. Paul, D. W. Bliss

Let (X, Y) be a pair of jointly continuous random variables with values over the space $\mathcal{X} \times \mathcal{Y}$. If their joint probability density function is $P_{(X,Y)}$ and the marginal probability density functions are P_X and P_Y, the mutual information is defined as:

$$I(X, Y) = \int_{\mathcal{Y}} \int_{\mathcal{X}} P_{(X,Y)}(x, y) \log\left(\frac{P_{(X,Y)}(x, y)}{P_X(x) P_Y(y)}\right) dxdy$$

Mutual information measures the information that X and Y share : it measures how much knowing one of these variables reduces uncertainty about the other. For example, if X and Y are independent, then knowing X does not give any information about Y and vice versa, so their mutual information is zero.

An information theoretic approach for performance metrics

A note on mutual information

Let's see some interpretation for the mutual information I in our case:

$$I(\boldsymbol{\theta} + \boldsymbol{N}_{est}; \boldsymbol{\theta})$$

The idea is to consider the entropy of the random parameter being estimated and the entropy of the estimation uncertainty of this parameter. The target, in the form of entropy (uncertainty) with respect to the radar tracker, is seen as an uncooperative communicator. In this sense, we can view the desired information to be communicated as the target source entropy and the undesired entropy as the total channel noise (receiver and estimation noise).

An information theoretic approach for performance metrics

Setting used

To characterize the estimation rate, we consider that the setting is a multicarrier waveform used to sense a single target in the presence of interference from the JCAS network. Here are some notations:

- The waveform consists of N_c subcarriers, and the maximum burst duration of the radar excitation signal is N_s multicarrier symbols.
- A subset $S_{\text{rad}} \in \{0,1\}^{N_s \times N_c}$ of these resource elements are used for sensing, and we note $\text{SINR}_{m,n}$ the corresponding SINR on a resource element.

An information theoretic approach for performance metrics

Theorem : Sensing Capacity Bounds

1. Let J denote the Fisher Information of the estimation problem related to sensing. Then the sensing capacity is upper bounded as:

$$C_{rad} \leqslant C_{rad}^{UB} = \frac{1}{2T_{CPI}} log_2(|\boldsymbol{I} + \boldsymbol{Q}^{1/2}\boldsymbol{J}\boldsymbol{Q}^{1/2}|)$$

2. We can give upper and lower bounds for C_{rad}^{UB} as follows:

$$\frac{1}{2} log_2(1 + G \cdot \mathrm{SINR}_{\mathrm{rad}}) \leqslant T_{CPI}\, C_{rad}^{UB} \leqslant log_2\left(1 + \frac{1}{2} G \cdot \mathrm{SINR}_{\mathrm{rad}}\right)$$

where

$$\mathrm{SINR}_{\mathrm{rad}} = \sum_{(m,n)\in S_{rad}} \eta_{m,n} \mathrm{SINR}_{m,n}$$

and $\{\eta_{m,n}\}$, G are constants depending on waveform parameters and prior covariance for θ.

An information theoretic approach for performance metrics

A note on Fisher Information

Let $f(X;\theta)$ be the density function of X conditioned on the value of θ.
We define the score as the θ-derivative of the log of the likelihood function:

$$s(\theta) := \frac{\partial \log \mathcal{L}(\theta)}{\partial \theta} = \frac{\partial \log f(X;\theta)}{\partial \theta}$$

It can be shown that $\mathbb{E}\left[s(\theta)|\theta\right] = 0$.
We then define the Fisher Information as the variance of the score:

$$\mathcal{J}(\theta) = \mathbb{E}\left[\left(\frac{\partial \log f(X;\theta)}{\partial \theta}\right)^2 \middle| \theta\right] = \int_{\mathbb{R}} \left(\frac{\partial \log f(x;\theta)}{\partial \theta}\right)^2 f(x,\theta) dx$$

Informally, the Fisher information is a way of measuring the amount of information that an observable random variable X carries about an unknown parameter θ upon which the probability of X depends.

1. From the Cramér-Rao bound, we have:

$$\mathrm{Cov}(\boldsymbol{N}_{\mathrm{est}}) = \boldsymbol{R} \geqslant \boldsymbol{J}^{-1}$$

   where the inequality $\boldsymbol{A} \geqslant \boldsymbol{B}$ means that $\boldsymbol{A} - \boldsymbol{B}$ is semi-definite positive. Noticing that all the matrices are semi-definite positive, the result is then immediate.

2. We first compute the Fisher Information Matrix, which is a 2×2 matrix. Then, we deduce the following inequalities:

$$1 + Tr(\boldsymbol{Q}^{1/2}\boldsymbol{J}\boldsymbol{Q}^{1/2}) \leqslant |\boldsymbol{I} + \boldsymbol{Q}^{1/2}\boldsymbol{J}\boldsymbol{Q}^{1/2}| \leqslant \frac{1}{4} Tr\left(\boldsymbol{I} + \boldsymbol{Q}^{1/2}\boldsymbol{J}\boldsymbol{Q}^{1/2}\right)^2.$$

   The lower bound comes from the positive semi-definiteness of $\boldsymbol{Q}^{1/2}\boldsymbol{J}\boldsymbol{Q}^{1/2}$, and the upper bound can be deduced from the AM-GM inequality. We conclude the proof noting that $G \cdot \mathrm{SINR}_{\mathrm{rad}} = Tr(\boldsymbol{Q}^{1/2}\boldsymbol{J}\boldsymbol{Q}^{1/2})$.

An information theoretic approach for performance metrics

Performance metrics

From the previous slides, we see that $\mathsf{SINR}_{\mathsf{com}}$ and $\mathsf{SINR}_{\mathsf{rad}}$ can be used to characterize (or bound) the communication and sensing capacities. This leads us to define the following quantities:

- For coverage:

$$\mathbb{P}^0_{\Phi_U}(\mathsf{SINR}_{\mathsf{com}} \geqslant \tau) \text{ and } \mathbb{P}^0_{\Phi_S}(\mathsf{SINR}_{\mathsf{rad}} \geqslant \tau)$$

- For capacity:

$$\mathbb{E}^0_{\Phi_U}\left[log_2(1 + \mathsf{SINR}_{\mathsf{com}})\right] \text{ and } \mathbb{E}^0_{\Phi_S}\left[log_2(1 + G \cdot \mathsf{SINR}_{\mathsf{rad}})\right]$$

Models and assumptions for commmunication and sensing

Path loss functions

- For communication, the LoS and NLoS path loss functions are respectively:

$$g_L(r) = K_L r^{-\alpha_L} e^{-\gamma_L r} \text{ and } g_N(r) = K_N r^{-\alpha_N} e^{-\gamma_N r}$$

This gives the following path loss to the origin:

$$L(||X_k||_2) = g_L(||X_k||_2) M_k + g_N(||X_k||_2)(1 - M_k)$$

- For sensing, the two way path loss is:

$$g_{L,ret}(r) = \frac{K_L}{4\pi} r^{-\alpha_L} e^{-2\gamma_L r}$$

Models and assumptions for commmunication and sensing

Fading and cross section

- The typical UE faces Rayleigh fading on the desired signal :

$$|H^0|^2 \sim Exp(1)$$

- The radar cross section of the typical SO is constant and assumed exponentially distributed :

$$\kappa_{CS} \sim Exp(1)$$

- For interfering links, the fading terms are assumed IID and distributed as follows:

$$|H_n^k|^2 \sim \Gamma(N_L, N_L) \text{ when } X_k \text{ is LoS to the receiver,}$$

$$|H_n^k|^2 \sim \Gamma(N_N, N_N) \text{ when } X_k \text{ is NLoS to the receiver,}$$

where $N_L, N_N \geqslant 1$ denote the order of the fading model.

Models and assumptions for commmunication and sensing

Beamforming and antennas

- The BSs and UEs perform directional beamforming for both communication and sensing.
- For a communication link, the BS and UE select the beam directions which maximize the received power.
- Directionnal antennas are used for transmission and reception. Their antenna patterns both follow the sectored model:

$$G(\theta) = \begin{cases} G_{\max} & \text{when } \theta \in [\theta_0 \pm 3dB] \\ \xi \cdot G_{\max} & \text{otherwise.} \end{cases}$$

Models and assumptions for commmunication and sensing

UE and SO Association Policies

- The typical UE is associated with the BS that has optimal path loss. Denoting this BS by X_0, we have:

$$X_0 = \arg\sup\{L(||X_k||_2) : X_k \in \tilde{\Phi}_B\}$$

- The typical SO is associated with the nearest LoS BS. Denoting this BS by X_0, we have:

$$X_0 = \arg\inf\{||X_k||_2 : X_k \in \tilde{\Phi}_L\}$$

Models and assumptions for commmunication and sensing

SINR Models

Letting ν_{com} denote the normalized noise power at the typical UE, we can express SINR_{com} as:

$$\text{SINR}_{\text{com}} = \frac{|H^0|^2 L(||X_0||_2)}{\sum_{X_k \in \bar{\Phi}_B \setminus \{X_0\}} |H^k|^2 B^k Z_U^k L(||X_k||) + \nu_{com}}$$

Similarly, we can define an SINR for the sensing case:

$$\text{SINR}_{\text{rad}} = \mathbb{1}\{\Phi_L(\mathbb{R}^2) > 0\} \sum_{(t,n) \in \tilde{S}_{rad}} \theta_{t,n} \times \frac{\kappa_{CS} g_{L,ret}(||X_0||)}{\sum_{X_k \in \bar{\Phi}_B \setminus \{X_0\}} F_{t,n}^k Z_B^k L(||X_k - X_0||) + \nu_{rad}}$$

where $\tilde{S}_{rad}$ and $\{\theta_{t,n}\}$ are the reduction of S_{rad} and $\{\eta_{m,n}\}$ over time slots and subcarriers.

Analytical expressions and approximations

Step 1 : From distribution to Laplace Transform

For both SINR models, the fading term is exponentially distributed, meaning that their distribution can be characterized by the Laplace transform of the interference process:

$$I = \sum_{R_k \in \Phi} F_k g(R_k)$$

Where Φ is an arbitrary PPP with intensity measure Λ, $\{F_k\}$ are arbitrary fading terms and g is an arbitrary path loss function.

With the use of formula for the Laplace Transform of a PPP, this gives the following expression of the Laplace transform:

$$\mathcal{L}_I(s) = exp\left(-\int_{\mathbb{R}^+} (1 - \mathcal{L}_F(s \cdot g(r)))\Lambda(dr)\right)$$

Analytical expressions and approximations

Step 2 : From Laplace Transform to finite bounds

We define the Mellin Transform of the path loss with respect to Λ and restricted to a set A of $\mathbb{R}^+$ as:

$$\mathcal{M}_{(\Lambda \circ g^{-1})}(p; A) = \int_A g(r)^{p-1} \Lambda(dr)$$

Using these Mellin Transforms, we can then define two functions H_{LB} and H_{UB} which are finite support approximations of Λ. The Laplace transform can then be upper and lower bounded by some exponential of finite sums of these terms.

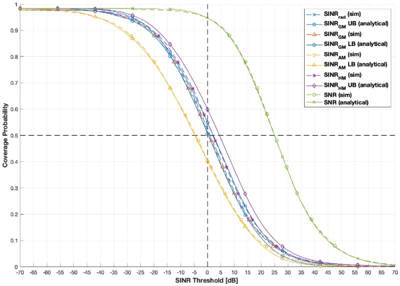
Analytical expressions and approximations

From finite bounds to tractable bounds

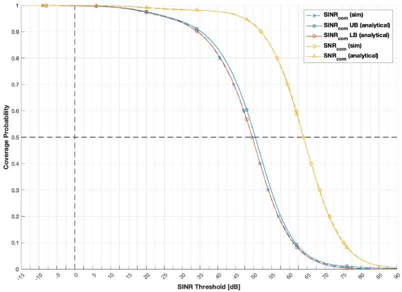
- For the intensity functions at stake, the associated path loss Mellin Transforms are not tractable. We then gives some bounds to them, which are expressed in terms of piecewise poly-exponential functions, allowing us to find closed forms for the Mellin Transforms.
- Those path loss Mellin Transforms can be expressed in closed forms using the generalized incomplete gamma function :

$$\Gamma(p, z_1, z_2) = \int_{z_1}^{z_2} x^{p-1} e^{-x} dx$$

- Thanks to that, we can give some computable bounds of the sensing coverage probability and communication coverage probability.

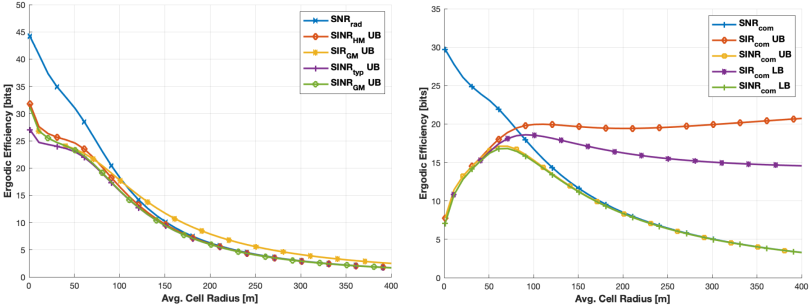


(a) Sensing



(b) Communication

Figure: Comparison of Coverage Probability for Sensing and Communication

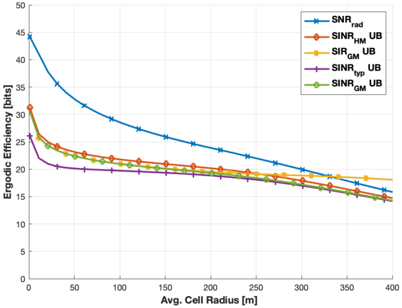


(a) Sensing (b) Communication

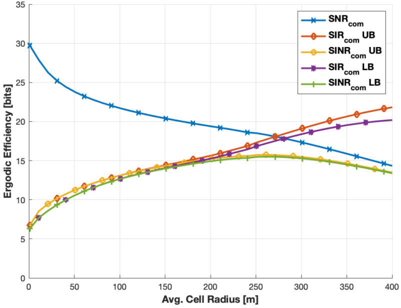
Figure: Estimation of the Ergodic Efficiency according to the Base Station Density in the low blockage regime

Numerical results

Ergodic Efficiency



(a) Sensing



(b) Communication

Figure: Estimation of the Ergodic Efficiency according to the Base Station Density in the high blockage regime

Conclusion

Conclusion

- Using an information theoretic framework, we have extended the notion of coverage probability to the radar setting, defining it as the probability that the rate of information associated with a typical sensing target exceeds some threshold.
- Focusing on the multicarrier setting, we established upper and lower bounds on the estimation rate in terms of SINR.
- We then used a stochastic geometry framework and some approximations on the model to obtain tractable bounds for the coverage probabilities and ergodic capacities.
- This leads us to notice for example that densification of the network improves sensing performance, in contrast to the communication function.

Conclusion

Future work

- Use of different network sensing approaches or different parameters of interest
- Impact of imperfect SO association with the serving BS (detection uncertainty, heterogeneity in BS capability)
- Other forms of BS cooperation or sensing policies among BSs, perhaps accounting for more structured deployments using non-Poisson spatial models.