Wireless communication channel for dummies (like me)

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Source: Tse, D., & Viswanath, P. (2005). Fundamentals of wireless communication. Cambridge university press.

Network theory reading group @LINCS

March 16, 2022

Goal for today

Understand from basic principles

- Where do complex signals come from
- Meaning of
  - “Channel” / “Channel response”
  - “Fading” (slow/fast/flat)
  - “Coherence time”
  - “Doppler spread”
  - “Delay spread”
  - “Coherence bandwidth”
  - “Inter-symbol interference”
  - “Baseband”

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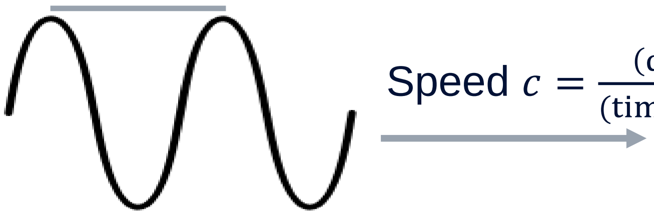
Wavelength, frequency and speed

$\lambda = c/f$



Speed $c = \dfrac{(\text{distance between two peaks})}{(\text{time interval between two peaks})} = \dfrac{\lambda}{T} = \lambda f$

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Free space, fixed rx antenna

Transmit electrical field:

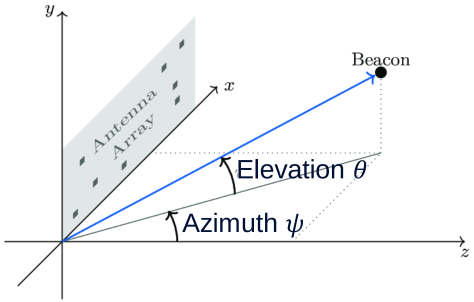
$$\alpha_s(\theta, \psi, f) \cos 2\pi f t$$

Receive: $\dfrac{\alpha_s(\theta,\psi,f) \cos 2\pi f\left(t-\frac{r}{c}\right)}{r}$

Distance r

Propagation delay $\frac{r}{c}$

$\alpha_s(\theta, \psi, f)$ is antenna gain in elevation θ and azimuth ψ



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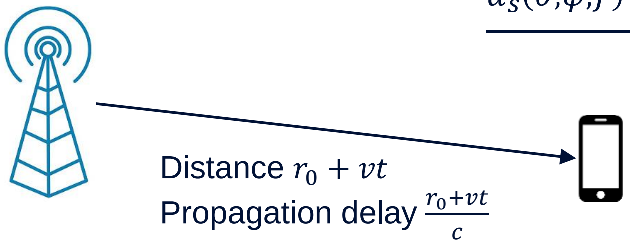
Free space, moving rx antenna

Transmit electrical field:
$\alpha_s(\theta, \psi, f)\cos 2\pi f t$

Receive: $\frac{\alpha_s(\theta,\psi,f)\cos 2\pi f\left(t-\frac{r_0+vt}{c}\right)}{r_0+vt} =$

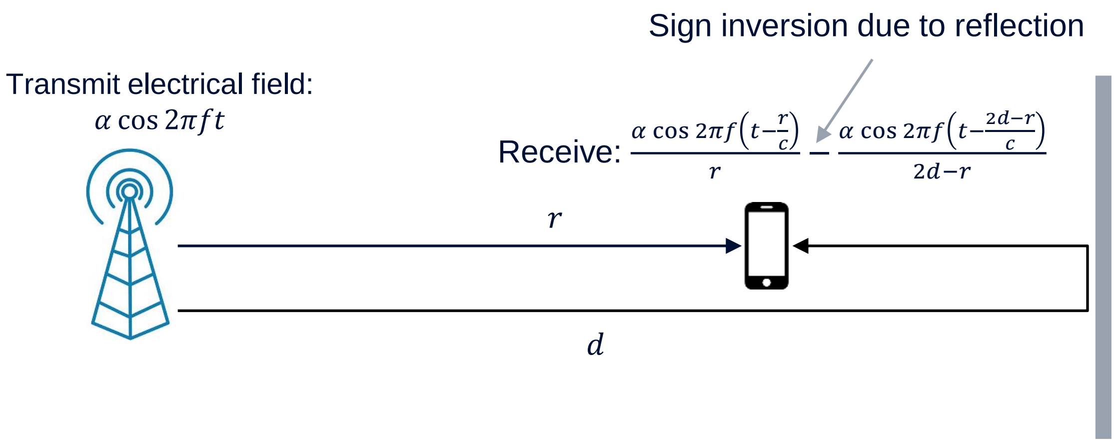
$\frac{\alpha_s(\theta,\psi,f)\cos 2\pi f\left(\left(1-\frac{v}{c}\right)t-\frac{r_0}{c}\right)}{r_0+vt} =$



Doppler shift: frequency decreases by factor $\left(1-\frac{v}{c}\right)$.

Note: the higher the frequency, the higher the shift

Reflecting wall, fixed rx antenna



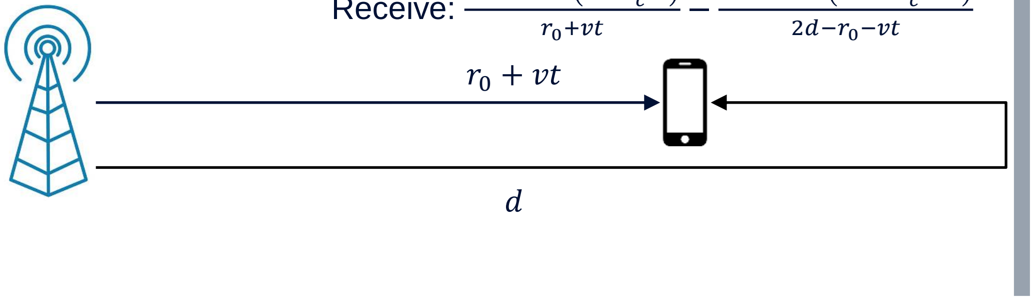
- Phase difference between two paths: $\Delta(r) = 2\pi\frac{r}{\lambda} - 2\pi\frac{2d-r}{\lambda} + \pi = 4\pi\frac{d-r}{\lambda} + \pi$
- If r changes by **coherence distance=$\boldsymbol{\lambda/4}$**, $\Delta(r)$ changes by π (constructive → destructive interference)
- **Delay spread $\boldsymbol{T_c}$** = time difference between propagation delays = $\frac{2d-r}{c} - \frac{r}{c}$
- **Coherence bandwidth:** If r stays fixed but frequency varies by $1/2T_c$, then Δ changes by π

Reflecting wall, moving rx antenna → Fading

Freq. $f\left(1-\frac{v}{c}\right)$ Freq. $f\left(1+\frac{v}{c}\right)$ → Doppler spread $=\frac{2fv}{c}\approx 50\text{Hz}$

Transmit electrical field:
$\alpha\cos 2\pi f t$

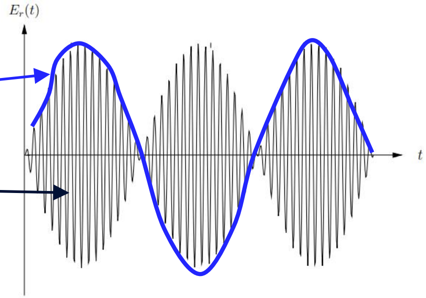
Receive: $\frac{\alpha\cos 2\pi f\left(t-\frac{r_0+vt}{c}\right)}{r_0+vt}-\frac{\alpha\cos 2\pi f\left(t-\frac{2d-r_0-vt}{c}\right)}{2d-r_0-vt}$



- Rx close to wall → Approximate two denominators as $= r_0 - vt$
- Receive: $\frac{2\alpha\sin 2\pi f\left[\frac{v}{c}t+\frac{r_0-d}{c}\right]\sin 2\pi f\left[t-\frac{d}{c}\right]}{r_0-vt}$

Slow – Doppler_spread/2: **Fading**

Fast – original frequency f



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Wireless channel as linear time invariant system

- Transmit $x(t)$ – in previous case, $x(t) = \alpha \cos 2\pi f t$
- Receive $y(t)$
- $y(t) = \sum_i a_i(t)\, x(t - \tau_i(t))$

Ex: For reflecting wall, moving antenna:

$a_1(t) = \frac{\alpha}{r_0+vt},\ a_2(t) = \frac{\alpha}{2d - r_0 - vt}$ path attenuation

$\tau_1(t) = \frac{r_0+vt}{c},\ \tau_2(t) = \frac{2d-r_0-vt}{c} - \frac{1}{2f}$ path delay

- We can rewrite above as convolution between input and channel response:

$$y(t) = \int h(\tau, t) x(t - \tau)$$

$$\text{where } h(\tau, t) = \sum_I a_i(t) = \delta\big(\tau - \tau_i(t)\big)$$

$$\text{in frequency domain: } H(f; t) = \sum_i a_i(t) e^{-j2\pi f \tau_i(t)}$$

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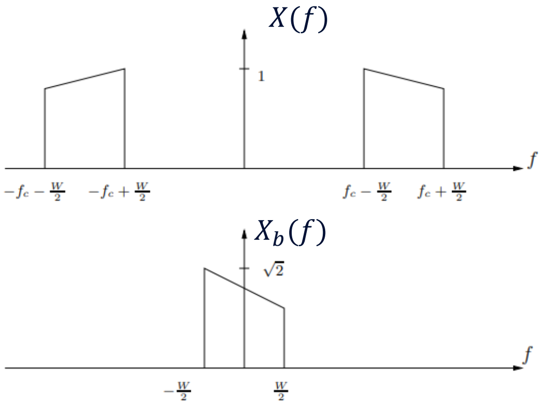
Baseband equivalent model

Move a step closer to reality: Transmit something different from a pure sinusoid



Fourier transform of "**over-the-air**" signal $X(f)$:

- Frequency centered at f_c
- Real → $X(-f) = X^*(f)$

Fourier transform of "**baseband**" signal $X_b(f)$:

- Frequency centered at 0
- Imaginary

$$\sqrt{2}X(f) = X_b(f - f_c) + X_b^*(-f - f_c)$$

inverse Fourier transform

$$x(t) = \frac{1}{\sqrt{2}}\left[x_b(t)e^{j2\pi f_c t} + x_b^*(t)e^{-j2\pi f_c t}\right] = \sqrt{2}\mathrm{Re}\left[x_b(t)e^{j2\pi f_c t}\right]$$

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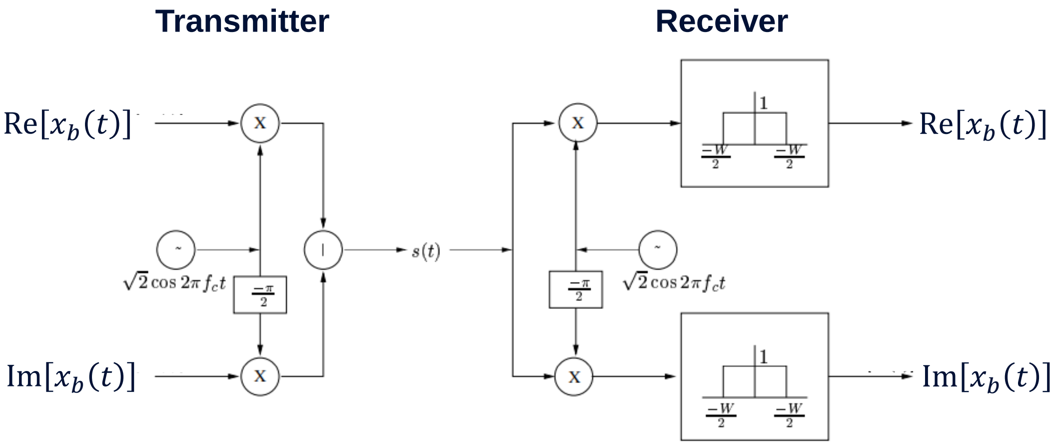
Baseband equivalent model

$$x(t) = \sqrt{2}\mathcal{R}\left[x_b(t)e^{j2\pi f_c t}\right] = \text{Re}[x_b(t)]\sqrt{2}\cos 2\pi f_c t - \text{Im}[x_b(t)]\sqrt{2}\sin 2\pi f_c t$$



Low-pass filter

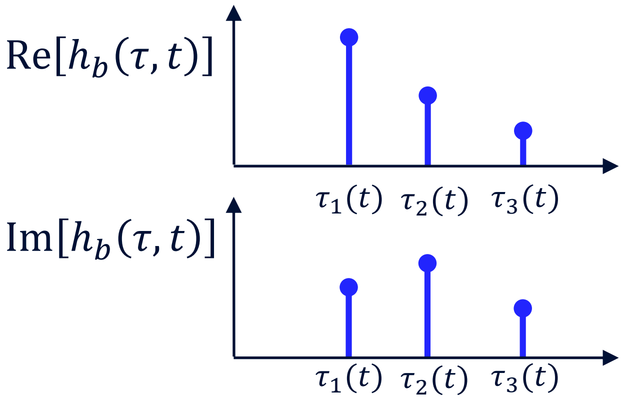
Baseband equivalent model

- Remember: $y(t) = \sum_i a_i(t)\, x(t - \tau_i(t))$
- Now, $x(t) = \sqrt{2}\mathrm{Re}\left[x_b(t) e^{j2\pi f_c t}\right]$
- We can prove that $y_b(t) = \sum_i a_i^b(t)\, x_b(t - \tau_i(t))$, where
  - $a_i^b(t) := a_i(t) e^{-j2\pi f_c \tau_i(t)} = a_i(t) e^{-j2\pi \frac{r_i}{\lambda}}$
  - $y(t) = \mathrm{Re}\left[y_b(t) e^{j2\pi f_c t}\right]$

Phase shift due to distance

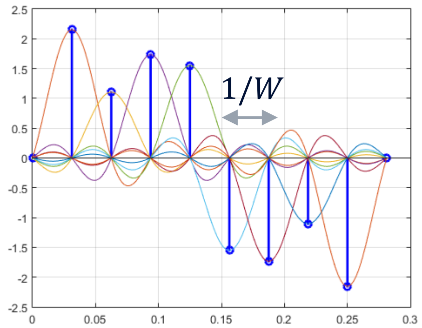
Path loss

- Baseband equivalent **impulse response**: $h_b(\tau, t) = \sum_i a_i^b(t) \delta\left(\tau - \tau_i(t)\right)$



Discrete-time baseband model

- Assume $x(t)$ has bandwidth W → $x_b(t)$ has bandwidth $W/2$ → we can sample $x_b(t)$ at frequency W w/o information loss (Shannon-Nyquist)
- $x_b(t) = \sum_n x_b[n]\,\mathrm{sinc}\,(Wt - n)$
  where $x_b[n] = x_b\left(\frac{n}{W}\right)$
  - Remember: $y_b(t) = \sum_i a_i^b(t)\, x_b(t - \tau_i(t))$
- Substitute and sample at $t = \frac{m}{W}$
  → $y_b[m] = \sum_n x_b[n] \sum_i a_i^b\left(\frac{m}{W}\right) \mathrm{sinc}\left(m - n - W\tau_i\left(\frac{t}{W}\right)\right)$
- $y_b[m] = \sum_\ell h^m[\ell]\, x[m-\ell]$ → inter-symbol interference
- Assume losses a and delays τ are time invariant:
  $h[\ell] = \sum_i a_i^b \mathrm{sinc}(\ell - \tau_i W) = (h_b \star \mathrm{sinc}(Wt))$ and sampled every $\frac{\ell}{W}$



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Fading: Time-varying channel (cfr slide 6)

- In the practice, losses a and delays τ do depend on time: $a(t), \tau(t)$
- In this case,$h^m[\ell] = \sum_i a_i\left(\frac{m}{W}\right) e^{-j2\pi f_c \tau_i\left(\frac{m}{W}\right)} \mathrm{sinc}\left(\ell - \tau_i\left(\frac{m}{W}\right) W\right)$
- How quickly does $h_\ell[m]$ vary over time?
  - $a_i(t)$ changes significantly over seconds (path loss)
  - Typically, $f_c \gg W$ → main factor for rapid (**phase**) changes is $e^{-j2\pi f_c \tau_i\left(\frac{m}{W}\right)}$, with speed $f_c \frac{d}{dt}\tau_i(t)$
  - The **magnitude** changes at time-scale of **coherence time** $T_c = \frac{1}{4D_s} \approx \mathrm{ms}$ inversely proportional to the **Doppler spread**:

$$D_s := \max_{i,j} f_c \left|\frac{d}{dt}\tau_i(t) - \frac{d}{dt}\tau_j(t)\right|$$

  - Cfr slide 6: $f\frac{d}{dt}\tau_i(t) = \pm\frac{fv}{c}$

Delay spread: Frequency-varying channel (cfr slide 5)

- **Delay spread:** Difference in propagation time between longest and shortest path:

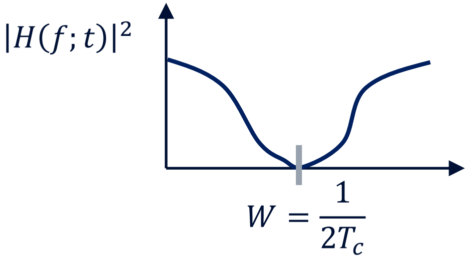
$$T_d := \max_{i,j} \left| \tau_i(t) - \tau_j(t) \right|$$

- How does this impact the behavior of channel response in *frequency*?

$$H(f;t) = \sum_i a_i(t) e^{-j2\pi f \tau_i(t)}$$

- Simple example: $a_1 = a_2 = 1, \tau_1 = 0, \tau_2 = \frac{1}{2W}$ → $T_c = \frac{1}{2W}$

$$|H(f;t)|^2 = \left|1 + e^{-\frac{j\pi f}{W}}\right|^2 = \left(1 + e^{-\frac{j\pi f}{W}}\right)\left(1 + e^{\frac{j\pi f}{W}}\right) = 2\left(1 + \cos\frac{\pi f}{W}\right)$$



In general, $H(f)$ changes "significantly" when f changes by **coherence bandwidth** $W_c = \frac{1}{2T_c}$

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The 4 main actors

Time	Frequency	Relationship
Coherence time T_c: time during which channel h can be considered as constant	**Doppler spread** D_s: Δf of incoming paths	$T_c = \frac{1}{4D_s}$
Delay spread T_d: max Δ propagation delay between two paths	**Coherence bandwidth:** bw over which channel $H(f)$ can be considered as constant	$W_c = \frac{1}{2T_d}$

Terminology

Type of channel	Characteristics
Fast fading	$T_c <$ delay requirements → coded symbol can be sent over different realizations of h
Slow fading	$T_c >$ delay requirements → coded symbol experience the same h
Flat fading	$W \ll W_c$ → signal “sees” a flat $H(f)$
Frequency-selective fading	$W \gg W_c$ → different frequencies of same signal are distorted in different ways
Under-spread	$T_d \gg T_c$ → h can be considered as constant over time

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Eliminate inter-symbol interference (ISI): OFDM

- How to go from $y[m] = \sum_\ell h_\ell[m]\, x[m-\ell]$ to $y[m] = h[m]x[m]$ (aka eliminate ISI)?