NOKIA Bell Labs

Multi-winner voting rules

François Durand

6 January 2021, Reading Group Network Theory of the Lincs

Voting rules in general

Voters, candidates (= options to choose from).

Input = ballots, typically:

- **Rankings** (complete or not, with ties or not),
- **Grades**,
- **Approvals**.

Output:

- **One candidate** (single-winner rules),
- **Several candidates** (multi-winner rules):
  - Committee of fixed size k (**this talk**),
  - Committee of variable size,
- **Ranking over the candidates** (social welfare functions).

Multi-winner voting rules: old and new problems

Problems that already exist in single-winner rules:

- **Condorcet paradox**,
- **Arrow theorem**,
- **Gibbard-Satterthwaite theorem**.

These problems still exist for multi-winner rules.

But we also have new problems:

- What **objective** do we pursue?
- **Computational complexity** of computing the winners.

Preliminary example

Candidates: $A_1, A_2, A_3, A_4, B_1, B_2, C_1, C_2$.

Voters	73	23	2	2
Approvals	A_1, A_2, A_3, A_4	B_1, B_2	C_1	D_1

Say we want to elect $k = 4$ candidates.
Who should win?

NOKIA Bell Labs

Preliminary example

Candidates: $A_1, A_2, A_3, A_4, B_1, B_2, C_1, C_2$.

Voters	73	23	2	2
Approvals	A_1, A_2, A_3, A_4	B_1, B_2	C_1	D_1

Say we want to elect $k = 4$ candidates.
Who should win?

Objective	**Example of scenario**	**Winners**
Excellence	Recruit $k = 4$ taxi drivers	$\{A_1, A_2, A_3, A_4\}$

Preliminary example

Candidates: $A_1, A_2, A_3, A_4, B_1, B_2, C_1, C_2$.

Voters	73	23	2	2
Approvals	A_1, A_2, A_3, A_4	B_1, B_2	C_1	D_1

Say we want to elect $k = 4$ candidates.
Who should win?

Objective	**Example of scenario**	**Winners**
Excellence	Recruit $k = 4$ taxi drivers	$\{A_1, A_2, A_3, A_4\}$
Proportionality	Elect a parliament of $k = 4$ members	$\{A_1, A_2, A_3, B_1\}$

Preliminary example

Candidates: $A_1, A_2, A_3, A_4, B_1, B_2, C_1, C_2$.

Voters	73	23	2	2
Approvals	A_1, A_2, A_3, A_4	B_1, B_2	C_1	D_1

Say we want to elect $k = 4$ candidates.
Who should win?

Objective	**Example of scenario**	**Winners**
Excellence	Recruit $k = 4$ taxi drivers	$\{A_1, A_2, A_3, A_4\}$
Proportionality	Elect a parliament of $k = 4$ members	$\{A_1, A_2, A_3, B_1\}$
Diversity	Choose locations for $k = 4$ defibrillators	$\{A_1, B_1, C_1, D_1\}$

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

  Public NOKIA Bell Labs

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

Our running example

Voters	27	12	5	22	21	13
Rankings	A	C	C	D	E	E
	B	D	E	C	A	B
	C	E	D	B	B	C
	D	B	A	E	D	A
	E	A	B	A	C	D
Approvals	A, B, C, D	C	A, D, C, E	B, C, D, E	A, B, D, E	E

We want to elect a committee of size $k = 2$.

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

   NOKIA Bell Labs

Best-k Rules

Recipe

Take a single-winner voting rule that produces scores (or a ranking over the candidates).

Output the k candidates with the best scores.

  Public NOKIA Bell Labs

Single Non-Transferable Voting (SNTV)

Principle: best k candidates by Plurality.

Voters	27	12	5	22	21	13
Rankings	**A**	**C**	**C**	**D**	**E**	**E**
	B	D	E	C	A	B
	C	E	D	B	B	C
	D	B	A	E	D	A
	E	A	B	A	C	D

Example: score$(A) = 1 \times 27 = 27$.

Candidate	A	B	C	D	E
Score	**27**	0	17	22	**34**

Winning committee: $S = \{A, E\}$.

  Public NOKIA Bell Labs

Bloc voting

Principle: best k candidates by k-approval (reminder: we consider $k = 2$).

Voters	27	12	5	22	21	13
Rankings	**A**	**C**	**C**	**D**	**E**	**E**
	B	**D**	**E**	**C**	**A**	**B**
	C	E	D	B	B	C
	D	B	A	E	D	A
	E	A	B	A	C	D

Example: $\text{score}(A) = 1 \times 27 + 1 \times 21 = 48$.

Candidate	A	B	C	D	E
Score	**48**	**40**	39	34	39

Winning committee: $S = \{A, B\}$.

NOKIA Bell Labs

best-k Borda

Principle: best k candidates by Borda rule.

Voters	27	12	5	22	21	13
Rankings	A	C	C	D	E	E
	B	D	E	C	A	B
	C	E	D	B	B	C
	D	B	A	E	D	A
	E	A	B	A	C	D

Example: $\text{score}(A) = 4 \times 27 + 0 \times 12 + 1 \times 5 + 0 \times 22 + 3 \times 21 + 1 \times 13 = 189.$

Candidate	A	B	C	D	E
Score	189	**218**	**214**	182	197

Winning committee: $S = \{B, C\}$.

best-k Approval Voting

Principle: best k candidates by Approval Voting.

Voters	27	12	5	22	21	13
Approvals	A, B, C, D	C	A, C, D, E	B, C, D, E	A, B, D, E	E

Example: $\text{score}(A) = 1 \times 27 + 1 \times 5 + 1 \times 21 = 53.$

Candidate	A	B	C	D	E
Score	53	**70**	66	**75**	61

Winning committee: $S = \{B, D\}$.

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

Committee scoring rules

Recipe

Find a way to make each voter v assign a score to each possible committee S:

$$\mathrm{score}_v(S).$$

Output the committee with the best score.

N.B.: all the best-k rules seen before belong to this family. We have in this case:

$$\mathrm{score}_v(S) = \sum_{c \in S} \mathrm{score}_v(c).$$

Example on next slide...

  Public NOKIA Bell Labs

best-k Borda, seen as a committee scoring rule

Reminders:
- the winning committee was $S = \{B, C\}$,
- $\text{score}(S = \{B, C\}) = \text{score}(B) + \text{score}(C) = 432.$

Let us compute $\text{score}(S = \{B, C\})$ another way:

Voters	27	12	5	22	21	13
Rankings	A	**C**	**C**	D	E	E
	B	D	E	**C**	A	**B**
	C	E	D	**B**	**B**	**C**
	D	**B**	A	E	D	A
	E	A	**B**	A	**C**	D
$\text{score}_v(S)$	5	5	4	5	2	5
$\lvert v \rvert \cdot \text{score}_v(S)$	135	60	20	110	42	65

$\Rightarrow \text{score}(S = \{B, C\}) = 135 + 60 + 20 + 110 + 42 + 65 = 432.$

Proportional Approval Voting (PAV)

Principle: $\text{score}_v(S) = 1 + 1/2 + \ldots + 1/i$,
where i is the number of candidates in the committee S approved by voter v.

Winning committee: $S = \{C, D\}$ (believe me).
For the example, let us compute $\text{score}(S = \{C, D\})$:

Voters	27	12	5	22	21	13
Approvals	A, B, **C**, **D**	**C**	A, **D**, **C**, E	B, **C**, **D**, E	A, B, **D**, E	E
$\text{score}_v(S)$	1.5	1	1.5	1.5	1	0
$\lvert v\rvert \cdot \text{score}_v(S)$	40.5	12	7.5	33	21	0

$\Rightarrow \text{score}(S = \{C, D\}) = 40.5 + 12 + 7.5 + 33 + 21 + 0 = 114.$

Borda Chamberlin-Courant (a.k.a. just "Chamberlin-Courant")

Principle: $\text{score}_v(S) = \text{Borda}_v(c)$,
where c is the candidate that voter v likes best in the committee S.

Winning committee: $S = \{A, C\}$ (believe me).
For the example, let us compute $\text{score}(S = \{A, C\})$:

Voters	27	12	5	22	21	13
Rankings	**A**	**C**	**C**	D	E	E
	B	D	E	**C**	**A**	B
	C	E	D	B	B	**C**
	D	B	A	E	D	A
	E	A	B	A	C	D
$\text{score}_v(S)$	4	4	4	3	3	2
$\|v\| \cdot \text{score}_v(S)$	108	48	20	66	63	26

$\Rightarrow \text{score}(S = \{A, C\}) = 108 + 48 + 20 + 66 + 63 + 26 = 331.$

Approval Chamberlin-Courant (a.k.a. Approval-CC)

Principle: $\text{score}_v(S) = \text{Approval}_v(c)$,
where c is the candidate that voter v likes best in the committee S.

Winning committee: $S = \{C, E\}$ (believe me).
For the example, let us compute $\text{score}(S = \{C, E\})$:

Voters	27	12	5	22	21	13
Approvals	A, B, **C**, D	**C**	A, **C**, D, **E**	B, **C**, D, **E**	A, B, D, **E**	**E**
$\text{score}_v(S)$	1	1	1	1	1	1
$\lvert v \rvert \cdot \text{score}_v(S)$	27	12	5	22	21	13

$\Rightarrow \text{score}(S = \{C, E\}) = 27 + 12 + 5 + 22 + 21 + 13 = 100.$

Committee scoring rules: theory

$\text{score}_v(c) = ?$

- Plurality (SNTV),
- k-approval (Bloc),
- Borda (k-Borda, Borda-CC),
- Approval (best-k Approval, PAV, Approval-CC).

$\text{score}_v(S) = ?$

- $\sum_{c \in S} \text{score}_v(c)$ (best-k rules),
- $\sum_i \alpha_i \cdot \text{score}_v(c_i)$, where c_i is the i-th preferred candidate of v in S (PAV).
- $\max_{c \in S} \text{score}_v(c)$ (Chamberlin-Courant).

N.B.: all are particular cases of the second one, called **order-weighted average**.

$\text{score}(S) = \sum_v \text{score}_v(S)$ (but we could choose otherwise).

Committee scoring rules: sum-up table

$\text{score}_v(c) =$	$\text{score}_v(S) =$		
	$\text{sum}_{c \in S} \text{score}_v(c)$	$\sum_i \alpha_i \cdot \text{score}_v(c_i)$	$\max_{c \in S} \text{score}_v(c)$
Plurality	SNTV		
k-approval	Bloc		
Borda	best-k Borda		Borda-CC
Approval	best-k Approval	PAV	Approval-CC

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

Other rules

Not all multi-winner voting rules are committee scoring rules!

Iterated single-winner rules

- Elect one candidate by the single-winner rule.
- Remove her from the ballots and iterate.

Example: with Plurality.

Voters	27	12	5	22	21	13
Rankings	**A**	**C**	**C**	**D**	**E**	**E**
	B	D	E	C	A	B
	C	E	D	B	B	C
	D	B	A	E	D	A
	E	A	B	A	C	D

$\Rightarrow$ Winners $= \{\ \ ,\ \ \}$.

Iterated single-winner rules

- Elect one candidate by the single-winner rule.
- Remove her from the ballots and iterate.

Example: with Plurality.

Voters	27	12	5	22	21	13
Rankings	**A**	**C**	**C**	**D**		
	B	D		C	**A**	**B**
	C		D	B	B	C
	D	B	A		D	A
		A	B	A	C	D

$\Rightarrow$ Winners $= \{\ \ , E\}$.

Iterated single-winner rules

- Elect one candidate by the single-winner rule.
- Remove her from the ballots and iterate.

Example: with Plurality.

Voters	27	12	5	22	21	13
Rankings	**A**	**C**	**C**	**D**		
	B	D		C	**A**	**B**
	C		D	B	B	C
	D	B	A		D	A
		A	B	A	C	D

$\Rightarrow$ Winners $= \{A, E\}$.

  Public NOKIA Bell Labs

Single Transferable Vote (STV)

- $\text{Quota}_k = \frac{V}{k+1}$. Ex: $\text{Quota}_1 = 50$, $\text{Quota}_2 = 33.3$, $\text{Quota}_3 = 25$...
- Elect all candidates with more than Quota_k top-votes and remove Quota_k voters who vote for each of them (see below). Iterate.
- If no candidate has the quota, eliminate the candidate with least top-votes.

Voters	27	12	5	22	21	13
Rankings	**A**	**C**	**C**	**D**	**E**	**E**
	B	D	E	C	A	B
	C	E	D	B	B	C
	D	B	A	E	D	A
	E	A	B	A	C	D

$\Rightarrow$ Winners $= \{\ \ ,\ \ \}$.

Single Transferable Vote (STV)

- $\text{Quota}_k = \frac{V}{k+1}$. Ex: $\text{Quota}_1 = 50$, $\text{Quota}_2 = 33.3$, $\text{Quota}_3 = 25$...
- Elect all candidates with more than Quota_k top-votes and remove Quota_k voters who vote for each of them (see below). Iterate.
- If no candidate has the quota, eliminate the candidate with least top-votes.

Voters	27	12	5	22	0.41	0.25
Rankings	**A**	**C**	**C**	**D**		
	B	D		C	**A**	**B**
	C		D	B	B	C
	D	B	A		D	A
		A	B	A	C	D

$\Rightarrow$ Winners = { , E}.

  Public NOKIA Bell Labs

Single Transferable Vote (STV)

- $\text{Quota}_k = \frac{V}{k+1}$. Ex: $\text{Quota}_1 = 50$, $\text{Quota}_2 = 33.3$, $\text{Quota}_3 = 25$...
- Elect all candidates with more than Quota_k top-votes and remove Quota_k voters who vote for each of them (see below). Iterate.
- If no candidate has the quota, eliminate the candidate with least top-votes.

Voters	27	12	5	22	0.41	0.25
Rankings	**A**			**D**		
	B	**D**			**A**	**B**
			D	B	B	
	D	B	A		D	A
		A	B	A		D

$\Rightarrow$ Winners $= \{\ \ , E\}$.

Single Transferable Vote (STV)

- $\text{Quota}_k = \frac{V}{k+1}$. Ex: $\text{Quota}_1 = 50$, $\text{Quota}_2 = 33.3$, $\text{Quota}_3 = 25$...
- Elect all candidates with more than Quota_k top-votes and remove Quota_k voters who vote for each of them (see below). Iterate.
- If no candidate has the quota, eliminate the candidate with least top-votes.

Voters	27	12	5	22	0.41	0.25
Rankings	**A**			**D**		
	B	**D**			**A**	**B**
			D	B	B	
	D	B	A		D	A
		A	B	A		D

$\Rightarrow$ Winners $= \{D, E\}$.

  Public NOKIA Bell Labs

Condorcet rules

Principle: if there exists S of size k such that any candidate in S beats any candidate out of S, then S must be selected.

Weighted majority matrix of our example:

	A	B	C	D	E
A		**53**	48	**61**	27
B	47		**61**	**61**	49
C	**52**	39		**57**	**66**
D	39	39	43		**61**
E	**73**	**51**	34	39	

Here there is no such set S, because A $>_{Maj}$ B $>_{Maj}$ C $>_{Maj}$ D $>_{Maj}$ E $>_{Maj}$ A.
The winning committee will depend on the particular Condorcet rule we use (beyond the scope of this talk).

Borda Monroe (a.k.a. just “Monroe”)

Variant of Chamberlin-Courant ensuring that not too many voters are “represented” by the same candidate.
Beyond the scope of this talk.

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

A word on computational complexity

Not computable in polynomial time:

- PAV,
- Monroe (in general),
- Chamberlin-Courant (in general).

Sequential variant: start from $S = \varnothing$ and add candidates one by one greedily.

Reverse sequential variant: start from $S = \{\text{all the candidates}\}$ and remove candidates one by one greedily.

Other approaches: fixed-parameter tractability (FPT), heuristics.

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

New running example

Voters	66	12	11	10	1
Rankings	A_1 A_2 A_3 $\vdots$ B_1 $\vdots$ C_1 $\vdots$	B_1 B_2 B_3 $\vdots$ A_1 $\vdots$ C_1 $\vdots$	B_2 B_1 B_3 $\vdots$ A_1 $\vdots$ C_1 $\vdots$	B_3 B_2 B_1 $\vdots$ A_1 $\vdots$ C_1 $\vdots$	C_1 C_2 C_3 $\vdots$ A_1 $\vdots$ B_1 $\vdots$
Approvals	All A_i	All B_i	All B_i	All B_i	All C_i

Assumption: we want to elect $k = 3$ candidates.

Excellence

Intuition: select the "best" candidates based on some criterion.

$\Rightarrow$ An **individual** notion about each elected candidate (rather than a notion about the elected committee as a whole).

Examples:

Criterion	Voting rule
Number of approvals	best-k Approval
Borda score	best-k Borda
Being preferred by a majority of voters	Condorcet rules

Excellence: k-best Approval

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

Winners = any three A_i's (depending on the tie-breaking rule).

Rationale: each A_i is "better" than any non-A candidate, because more approved.

Excellence: Condorcet Rules

Voters	66	12	11	10	1
Rankings	A_1	B_1	B_2	B_3	C_1
	A_2	B_2	B_1	B_2	C_2
	A_3	B_3	B_3	B_1	C_3
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	B_1	A_1	A_1	A_1	A_1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	C_1	C_1	C_1	C_1	B_1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Approvals	All A_i	All B_i	All B_i	All B_i	All C_i

Winners $= \{A_1, A_2, A_3\}$.

Rationale: each of them is “better” than (= preferred by a majority to) any non-elected candidate.

Excellence: Concluding Remark

- The two rules in previous slides give (approximately) the same outcome.
- But for some other rules that can be defended as promoting “excellence”, the outcome could be different: for example, k-best Plurality would elect $\{A_1, B_1, B_2\}$.
- Excellence is **not** a formally defined notion.

  Public NOKIA Bell Labs

Proportionality

Intuition: more numerous voters should be “represented” by more candidates.

If voters and candidates can be partitioned into several (political) parties, such that all voters of a party prefers all candidates of their party to all other candidates, **then** each party should have a number of seats proportional to the number of voters in her party (up to roundings).

$\Rightarrow$ Proportionality **is** a formally defined notion that says what should be the outcome in some particular profiles (but not all of them).

Proportionality: Proportional Approval Voting (PAV)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

$$\Delta_S(A_i) = 66$$
$$\Delta_S(B_i) = 33$$
$$\Delta_S(C_i) = 1$$

$$\text{Winners} = \{\quad,\quad,\quad\}.$$

Proportionality: Proportional Approval Voting (PAV)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

$$\Delta_S(A_i) = 66$$
$$\Delta_S(B_i) = 33$$
$$\Delta_S(C_i) = 1$$

$\Rightarrow$ Elect A_1 (for example).

Winners $= \{A_1, \quad , \quad \}$.

Proportionality: Proportional Approval Voting (PAV)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

$$\left.\begin{aligned} \Delta_S(A_i) &= 66/2 = 33 \\ \Delta_S(B_i) &= 33 \\ \Delta_S(C_i) &= 1 \end{aligned}\right\}$$

Here is the trick that makes PAV proportional:
Adding a **second** A_i or a **first** B_i gives as many points.

$$\text{Winners} = \{A_1, \quad , \quad \}.$$

  Public NOKIA Bell Labs

Proportionality: Proportional Approval Voting (PAV)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

$$\left.\begin{array}{l} \Delta_S(A_i) = 66/2 = 33 \\ \Delta_S(B_i) = 33 \\ \Delta_S(C_i) = 1 \end{array}\right\}$$

Here is the trick that makes PAV proportional:
Adding a **second** A_i or a **first** B_i gives as many points.

$\Rightarrow$ Elect A_2 (for example).

Winners $= \{A_1, A_2, \quad\}$.

Proportionality: Proportional Approval Voting (PAV)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

$$\Delta_S(A_i) = 66/3 = 22$$
$$\Delta_S(B_i) = 33$$
$$\Delta_S(C_i) = 1$$

$$\text{Winners} = \{A_1, A_2, \quad \}.$$

  Public NOKIA Bell Labs

Proportionality: Proportional Approval Voting (PAV)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

$$\Delta_S(A_i) = 66/3 = 22$$
$$\Delta_S(B_i) = 33$$
$$\Delta_S(C_i) = 1$$

$\Rightarrow$ Elect B_1 (for example).

Winners $= \{A_1, A_2, B_1\}$.

Proportionality: Proportional Approval Voting (PAV)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

$$\Delta_S(A_i) = 66/3 = 22$$
$$\Delta_S(B_i) = 33$$
$$\Delta_S(C_i) = 1$$

$\Rightarrow$ Elect B_1 (for example).

Winners $= \{A_1, A_2, B_1\}$.

For $k = 6$, we would have 4 A_i's and 2 B_i's because:

$$\Delta_S(\text{fourth } A_i) = 66/4 \; = \; \Delta_S(\text{second } B_i) = 33/2.$$

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	66	12	11	10	1
Rankings	$\mathbf{A_1}$	$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	$\mathbf{C_1}$
	A_2	B_2	B_1	B_2	C_2
	A_3	B_3	B_3	B_1	C_3
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	B_1	A_1	A_1	A_1	A_1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	C_1	C_1	C_1	C_1	B_1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\text{Winners} = \{\quad,\quad,\quad\}.$

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	41	12	11	10	1
Rankings		$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	$\mathbf{C_1}$
	$\mathbf{A_2}$	B_2	B_1	B_2	C_2
	A_3	B_3	B_3	B_1	C_3
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	B_1	A_2	A_2	A_2	A_2
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	C_1	C_1	C_1	C_1	B_1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\text{Winners} = \{A_1, \quad , \quad \}.$

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	16	12	11	10	1
Rankings		$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	$\mathbf{C_1}$
		B_2	B_1	B_2	C_2
	$\mathbf{A_3}$	B_3	B_3	B_1	C_3
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	B_1	A_3	A_3	A_3	A_3
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	C_1	C_1	C_1	C_1	B_1
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

$\text{Winners} = \{A_1, A_2, \quad\}.$

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	16	12	11	10	1
Rankings		$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	$\mathbf{C_1}$
		B_2	B_1	B_2	
	$\mathbf{A_3}$	B_3	B_3	B_1	
	B_1	A_3	A_3	A_3	A_3
	$\vdots$				
	C_1	C_1	C_1	C_1	B_1
					$\vdots$

$\text{Winners} = \{A_1, A_2, \quad \}.$

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	16	12	11	10	1
Rankings		$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	
		B_2	B_1	B_2	
	$\mathbf{A_3}$	B_3	B_3	B_1	
	B_1	A_3	A_3	A_3	$\mathbf{A_3}$
	$\vdots$				
					B_1
					$\vdots$

$\text{Winners} = \{A_1, A_2, \quad \}.$

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	16	12	11	10	1
Rankings	$\mathbf{A_3}$	$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_2}$	$\mathbf{A_3}$
	B_1	B_2	B_1	B_1	B_1
	$\vdots$	A_3	A_3	A_3	$\vdots$

Winners $= \{A_1, A_2, \quad \}$.

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	16	12	11	10	1
			$\mathbf{B_2}$		
		B_2		$\mathbf{B_2}$	
	$\mathbf{A_3}$				
Rankings	B_2	A_3	A_3	A_3	$\mathbf{A_3}$
					B_2

$\text{Winners} = \{A_1, A_2, \quad \}.$

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	16	12	11	10	1
Rankings	$\mathbf{A_3}$	B_2	$\mathbf{B_2}$	$\mathbf{B_2}$	$\mathbf{A_3}$
	B_2	A_3	A_3	A_3	B_2

Winners $= \{A_1, A_2, B_2\}$.

Proportionality: Single Transferable Vote (STV)

$k = 3 \Rightarrow \text{Quota}_k = \frac{100}{3+1} = 25.$

Voters	16	12	11	10	1
Rankings			$\mathbf{B_2}$		
		B_2		$\mathbf{B_2}$	
	$\mathbf{A_3}$				
	B_2	A_3	A_3	A_3	$\mathbf{A_3}$
					B_2

Winners $= \{A_1, A_2, B_2\}$.

For $k = 6$, we would have $\{A_1, A_2, A_3, A_4, B_2, B_1\}$.

Diversity

Intuition: as many voters as possible should be well “represented” by at least one candidate.

This is **not** a formally defined notion.

Diversity: Approval Chamberlin-Courant (Approval-CC)

Voters	66	33	1
Approvals	All A_i	All B_i	All C_i

Winners $= \{\text{any } A_i, \text{any } B_i, \text{any } C_i\}$.

Two possible rationales:

- Once A-voters have one candidate A_i in the outcome, they are as happy as they can be.
- **Or** they could be more happy, but it is more important to represent as many voters as possible, including the only C-voter.

Diversity: Concluding Remark

Classic example to justify diversity: choosing movies for the catalogue of a short plane travel, because each passenger will watch only one movie. But...

Assume the following poll result for a sample of potential passengers:

Voters	54.4%	27.2%	18.1%	0.1%	0.1%	0.1%
Approvals	Genre A	Genre B	Genre C	Genre D	Genre E	Genre F

For $k = 6$, do you really want:

- One movie of each genre?
- Or give at least two possible choices for the people who like genre A?

$\Rightarrow$ Diversity is a very extreme point of view, giving a big power to arbitrary small minorities.

Summary: Which rule for which objective?

Arguably:

- **Excellence** (select “good” candidates):
  Best-k rules, iterated single-winner rules, Condorcet rules.
- **Proportionality** (more voters should be represented by more candidates):
  PAV, STV, Monroe.
- **Diversity** (as many voters as possible should be represented):
  Borda-CC, Approval-CC.

In fact, since excellence and diversity are not formally defined, there are no clear frontiers between these three objectives...

Plan

Zoology of rules

- Best-k rules
- Committee scoring rules
- Other rules

Discussion

- A word on computational complexity
- Which rule for which objective?

Conclusion

Take-aways

- Multi-winner rules differ on their objective: **excellence**, **proportionality** or **diversity**.
- A large class of rules is given by the **committee scoring rules**.
- Some interesting rules are **computationally hard to compute**.

Bibliography: P. Faliszewski, P. Skowron, A. Slinko, and N. Talmon. Multiwinner voting: A new challenge for social choice theory. In U. Endriss, editor, Trends in Computational Social Choice. AI Access, 2017.

Thanks For Your Attention!

NOKIA