NOKIA Bell Labs

Coalitional Manipulation of Voting Rules: Simulations on Empirical Data

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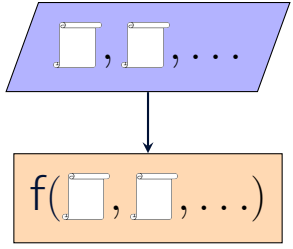
Invited paper in Constitutional Political Economy (preprint)

Lincs Seminar, Palaiseau, 11 January 2023

What are voting rules? And coalitional manipulability?

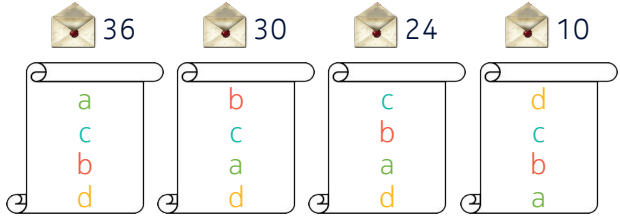
Voting rule: take ballots as input, yield one candidate (winner) as output.



Coalitional Manipulability (CM): some voters may deviate from their sincere ballots and get a winner they prefer.

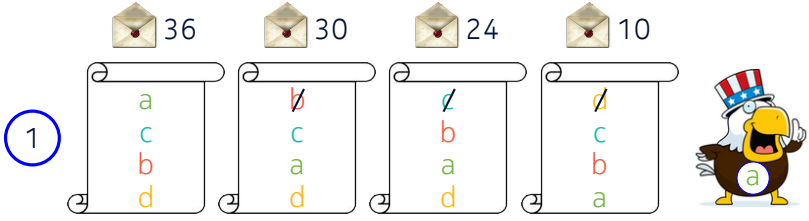
It will be clearer with an example of **profile**...

Example of profile



Plurality

Elect the candidate with most top-votes.



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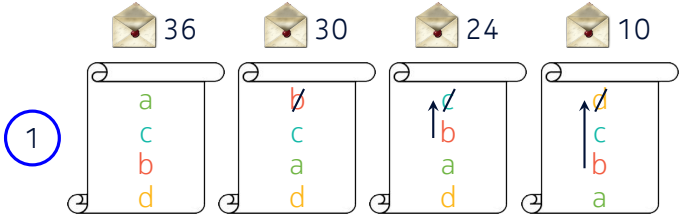
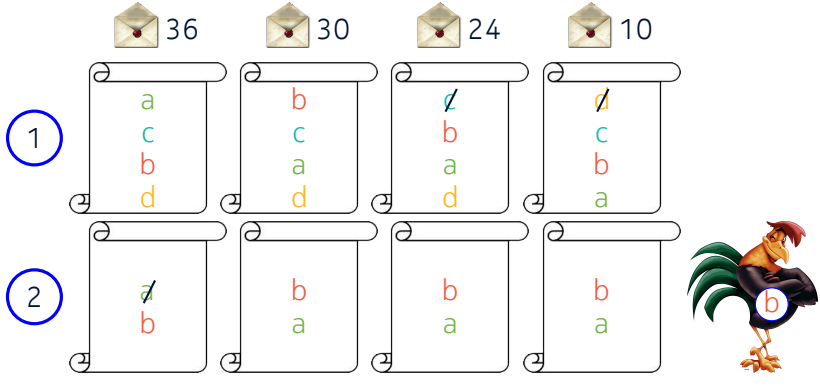


CM for b!

Two-round system

Keep the two candidates with most top-votes and retry.



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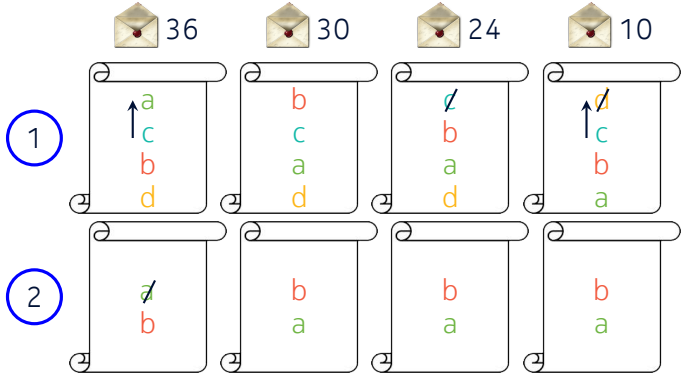
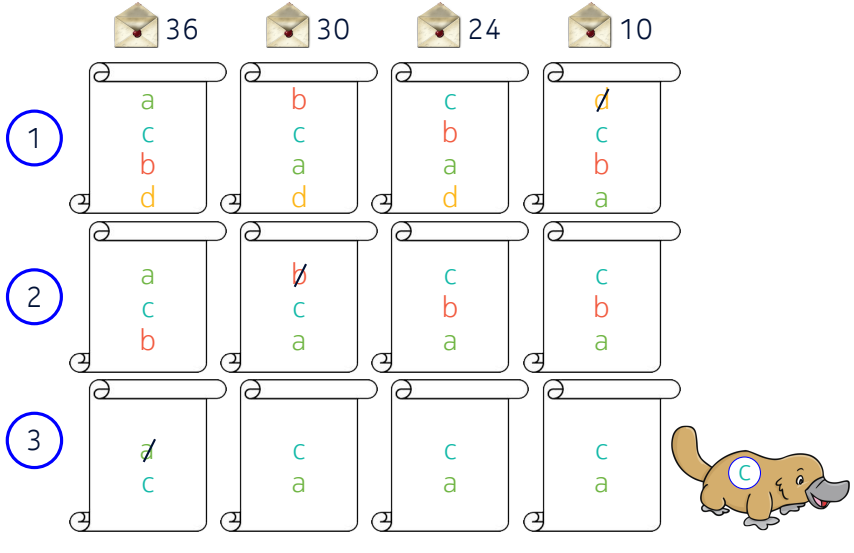


CM for c!

Instant Runoff Voting (IRV)

Recursively remove the candidate with least top-votes.



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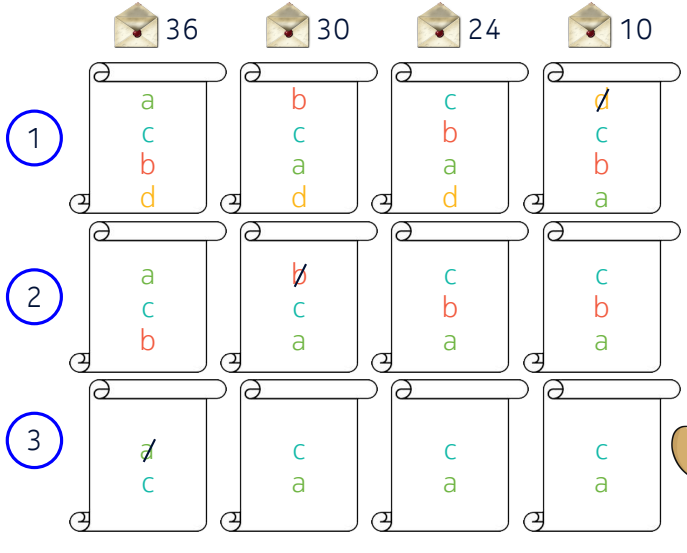
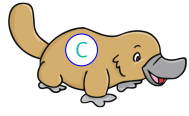
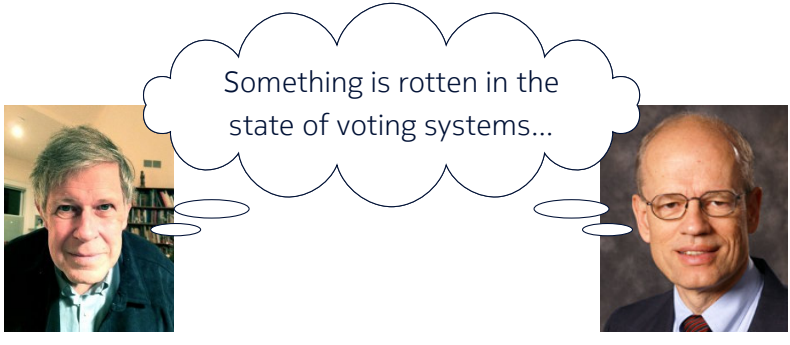




Not CM!
(Believe me)

The Gibbard-Satterthwaite Theorem (1973)

If a voting rule is **not dictatorial** and can elect **at least three different candidates**, then it is **manipulable** = there exists at least one profile where it is manipulable.



⇒ For non-trivial voting rules, we cannot rule out manipulability. We can just try to **minimize its scope**.

Plan

Voting Rules

Experimental Setup

Results

Conclusion

Grade-based voting systems

Range voting (RV)

Approval voting (AV)

Grade-based voting systems

Range voting (RV)

Approval voting (AV)

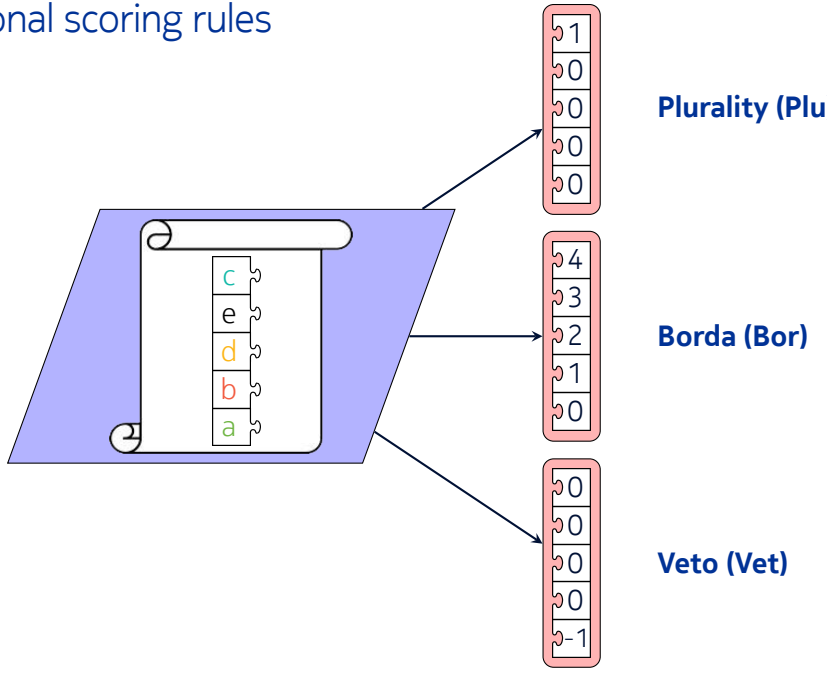
Grade-based voting systems

Range voting (RV)

Approval voting (AV)

Also: Majority Judgment (MJ), Scoring then Automatic Runoff (Star).

Positional scoring rules



Plurality (Plu)

Borda (Bor)

Veto (Vet)

Elimination methods

Principle: eliminate one or several candidates, then iterate.

- **Two-round system (TR):** eliminate all candidates but two (actual rounds).
- **Exhaustive ballot (EB):** eliminate one by one (actual rounds).
- **Instant-Runoff Voting (IRV):** eliminate one by one (virtual rounds).

Also: Baldwin (Bal), Nanson (Nan), Coombs (Coo), Kim-Roush (KR), Viennot (Vie).

Condorcet methods

Weighted majority matrix:

	a	b	c	d
a	—	**58**	**66**	**61**
b	42	—	**59**	**64**
c	34	41	—	**61**
d	39	36	39	—

Condorcet methods

a is **Condorcet winner**!

Weighted majority matrix:

	a	b	c	d
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Condorcet methods

a is **Condorcet winner**!

There is even a **Condorcet order**!

Weighted majority matrix:

	a	b	c	d
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Condorcet methods

Zut et flûte, no Condorcet winner!

Weighted majority matrix:

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Smith set = {a, b, c}...

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Rules: Copeland (Cop), Maximin (Max), Black (Bla), Ranked Pairs (RP), Schulze (Sch), Split Cycle (SC).

Condorcet variants of IRV

For normal people: mix IRV and Condorcet (and you can sleep until next slide).

For voting rule nerds:

- **Condorcet-IRV (CI):** If a Condorcet winner exists, elect her. Otherwise, elect the IRV winner.
- **Benham (Ben):** As long as the profile has no Condorcet winner, eliminate the candidate with the lowest plurality score. Then elect the Condorcet winner of the restricted profile.
- **Tideman (Tid):** Alternately, eliminate all the candidates outside the Smith set (if any), and the candidate with the lowest plurality score. When only one candidate remains, she is declared the winner.
- **Smith-IRV (SI):** Eliminate the candidates outside the Smith set, then run IRV on the restricted profile.
- **Woodall rule (Woo):** Among the candidates of the Smith set, elect the one that is eliminated latest in IRV.

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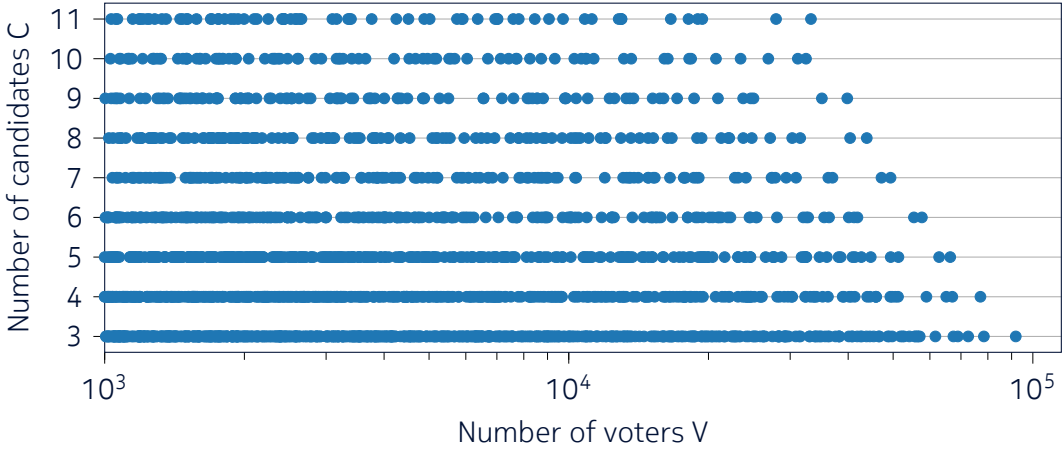
Datasets

In this talk: **“Netflix Prize”** dataset.

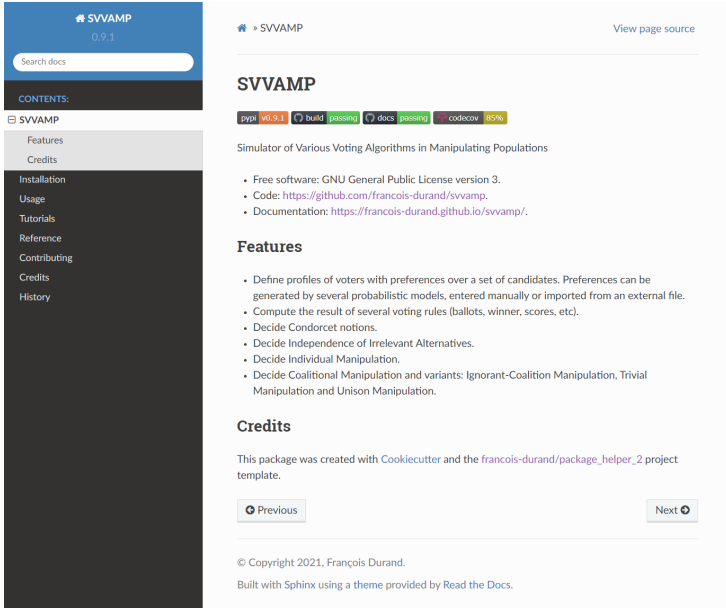
- **Users** (voters) assign **grades** to **movies** (candidates).
- Very **sparse** matrix.
- We extract **2,243 full matrices** (where each user rates each movie), with various number of users (voters) and movies (candidates).

In the paper, we also use another dataset: **US elections** with ranked ballots (member of city council, member of board of supervisors, mayor, sheriff, district attorney, school director, assessor treasurer, etc).
(Almost) excluded from this talk, but the results are similar.

Overview of the profiles



SVVAMP (https://github.com/francois-durand/svvamp)



Algorithms used to determine CM

And their time complexity

Voting rule	Algorithm
AV, MJ, Plu, RV, Star, TR, Vet, Buc	Exact (polynomial)
Bor	Approximate (polynomial) [Zuckerman et al., 2009].
Max	Approximate (polynomial) [Zuckerman et al., 2011].
Sch	Approximate (polynomial) [Gaspers et al., 2013].
EB	Exact (2^C), adapted from Coleman and Teague [2007].
Coo, IRV	Exact (C!), adapted from Coleman and Teague [2007].
Bal, Bla, Cop, KR, Nan, RP, SC, Vie	Heuristic (polynomial).
Ben, CI, SI, Tid, Woo	Heuristic (C!).

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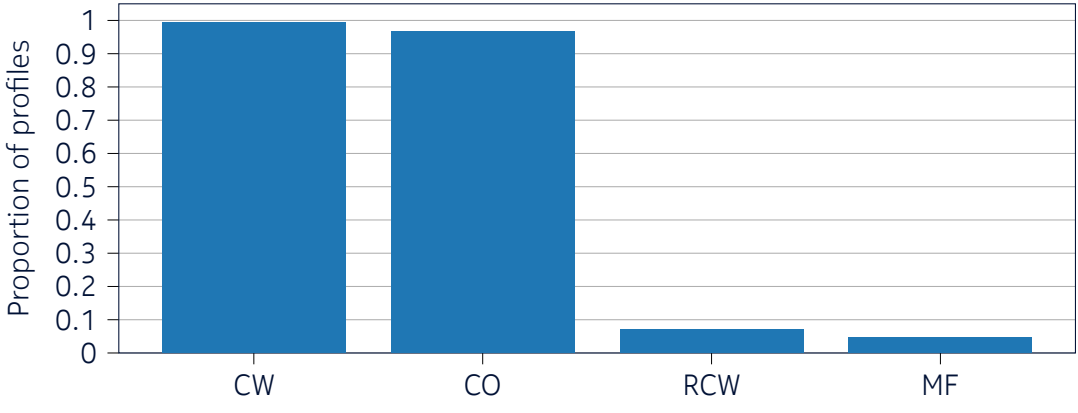
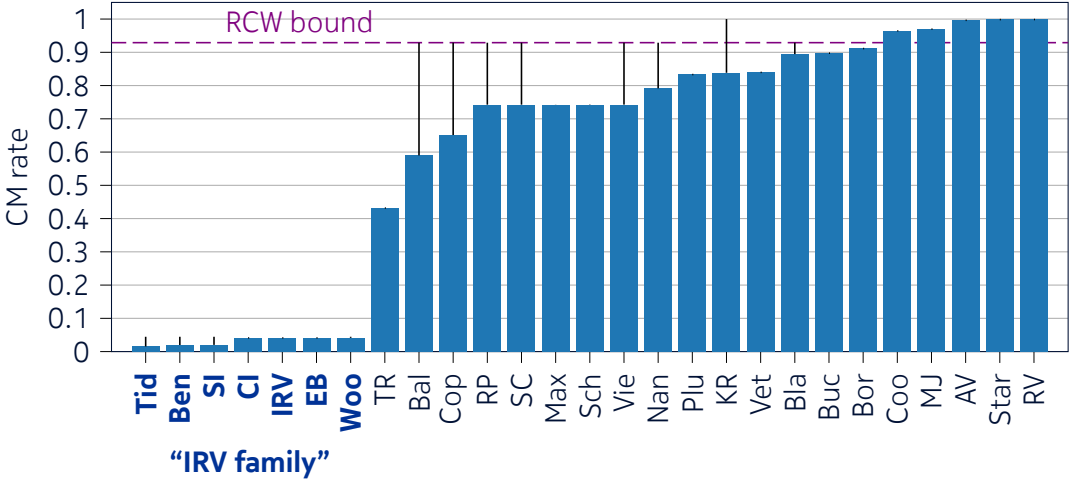
Qualitative features of the profiles



Condorcet Winner: majority in each pairwise comparison.
Condorcet Order: a candidate wins $C-1$ pairwise comparisons, another $C-2$, etc.
Resistant Condorcet Winner: majority ($> \frac{V}{2}$) in each 3-candidate comparison.
Majority Favorite: majority ($> \frac{V}{2}$) in the C-candidate comparison.

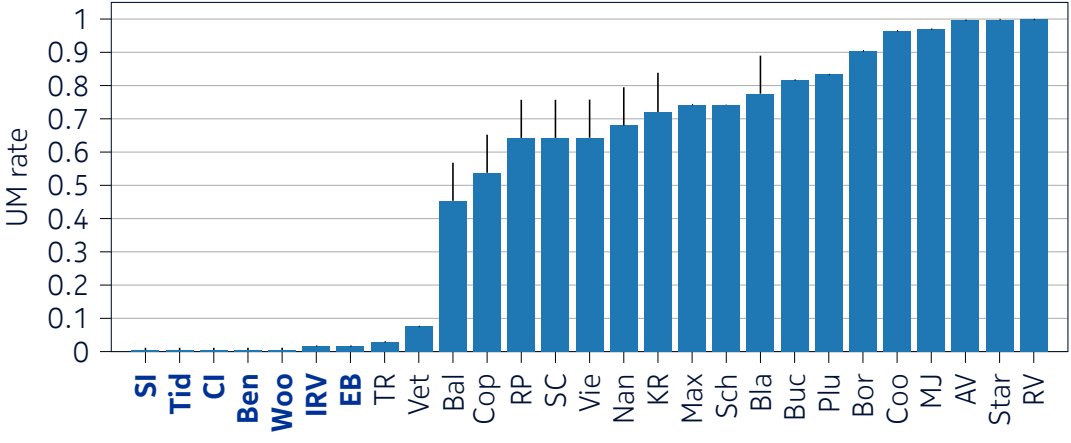
CM rate

Proportion of profiles that are coalitionally manipulable



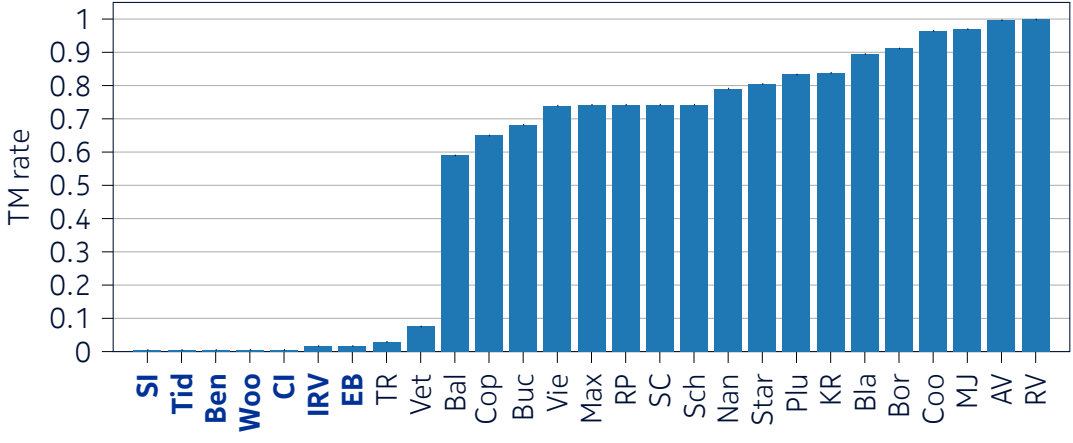
UM rate

Proportion of profiles that are unison manipulable (= all manipulators use the same strategic ballot)



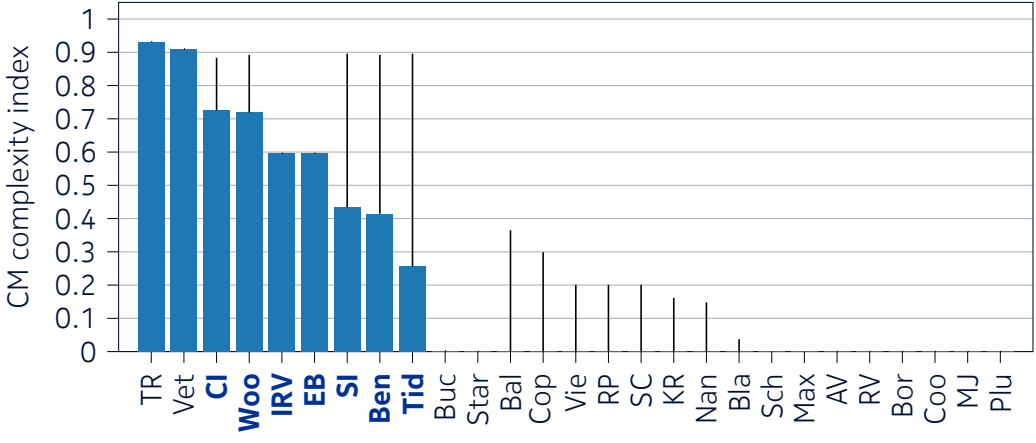
TM rate

Proportion of profiles that are trivially manipulable (= with a simple heuristic)



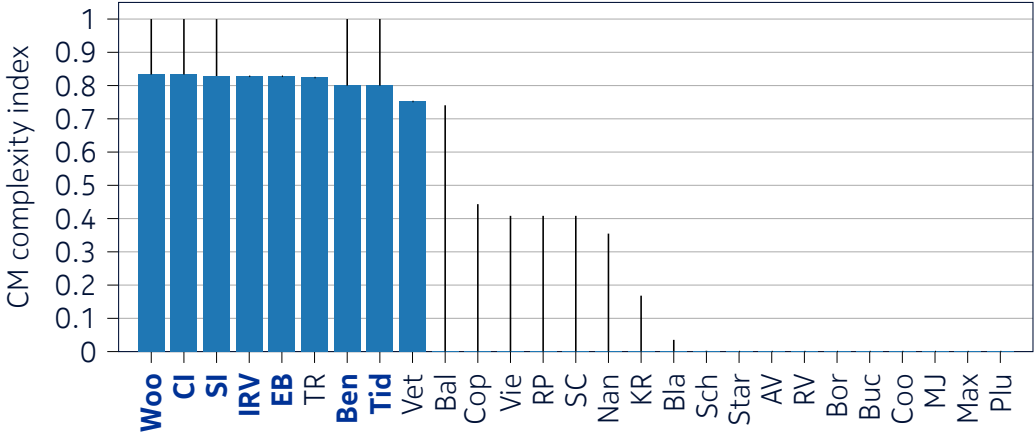
CM complexity index

$\mathbb{P}$(The profile is neither UM nor TM | The profile is CM)



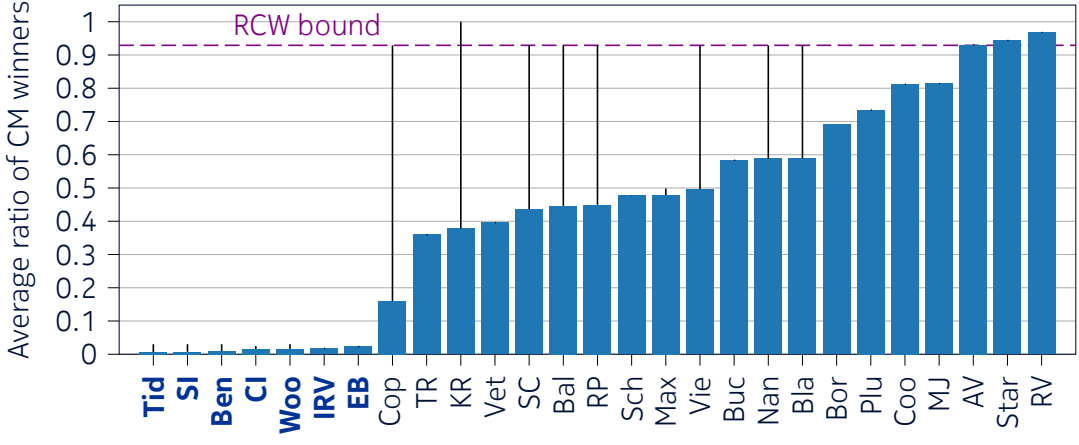
CM complexity index (for the FairVote dataset)

$\mathbb{P}$(The profile is neither UM nor TM | The profile is CM)



Average ratio of CM winners

$\mathbb{E}$(Proportion of candidates who are not the natural winner but can win by coalitional manipulation)



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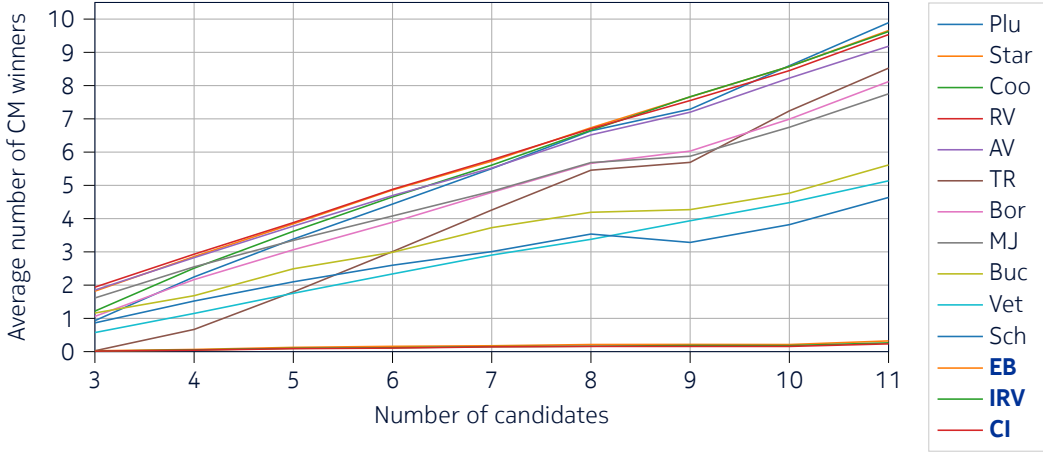
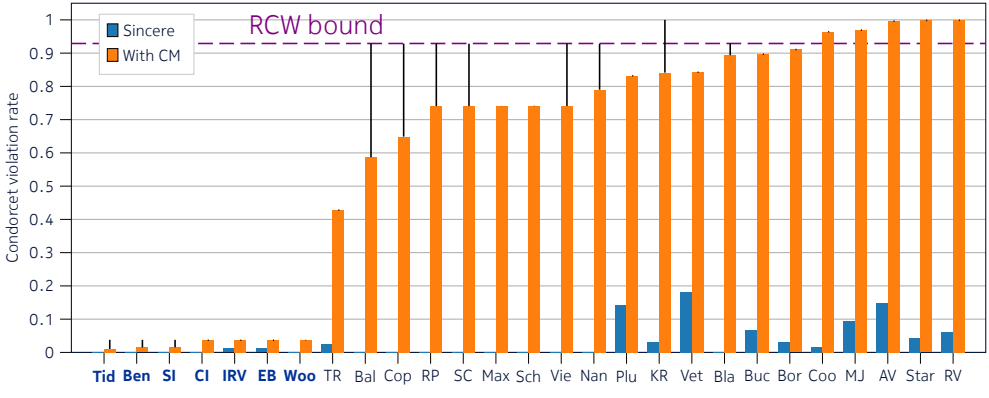


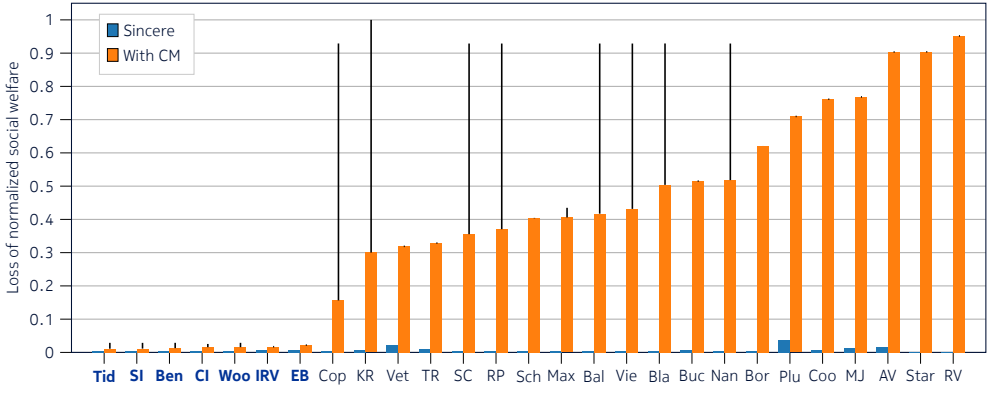
Condorcet violation rate

$\mathbb{P}$(The Condorcet winner is not elected | $\exists$ Condorcet winner)



Loss of normalized social welfare

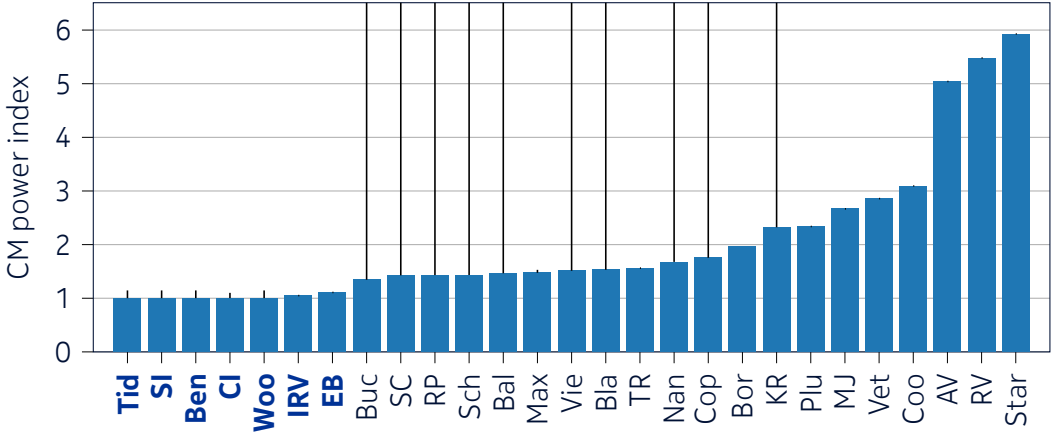
$\mathbb{E}$((Max total grade − Total grade of the winner) /(Max total grade − Min total grade))



CM power index

$\mathbb{E}(\max_{c \neq \text{natural winner}}(\text{Number of sincere voters / Minimal number of manipulators needed}))$

Intuitively: CM power index = X ⇔ A strategic voter has X times as much power as a sincere voter.



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Take-away

IRV and its variants are **more resilient to coalitional manipulability**, i.e. strategic voting, than all the other voting rules studied here, for all the indicators we considered.

The **differences** between the rules of the **IRV family** seem **at most marginal**.

- By theory, we know that: CM rate(CI) < CM rate(IRV) < CM rate(EB).
- For the other rules of the family, more precise algorithms (or theoretical results) would be needed to evaluate their respective performances.

“Future” work

Improve the CM algorithms to reduce the uncertainty margins. But:

- Version of SVVAMP used for this article: **0.8.3**.
- Version under development: **0.10.1**.

New features:

- **Improved CM algorithms**: Baldwin, Copeland, Kemeny, Kim-Roush, Nanson, Ranked Pairs, Split Cycle, Viennot.
- **New rules**: k-Approval and Slater.

Thanks For Your Attention!


NOKIA